A Stronger Duflo?

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8:46 AM

Question. Is the graded version of Duflo equivalent to the standard version?

$$\begin{tikzcd}
S(\mathfrak{g})[\hbar] \arrow[r, "WX^{\hbar}"] \arrow[d, "\hbar \mapsto 1"'] & U^{\hbar}(\mathfrak{g}) := T(\mathfrak{g})[\hbar] / xy-yx=\hbar[x,y] \arrow[d, "\hbar \mapsto 1"] \\
S(\mathfrak{g}) \arrow[r, "WX", "\text{homo by Duflo}"'] & U(\mathfrak{g})
\end{tikzcd}$$

Claim(?)

$$U^{\hbar}[\mathfrak{g}] \cong U(\mathfrak{g})[\hbar]$$

Duflo says that χ is a homomorphism. Does it follow that $\chi^{\hbar}$ is also a homomorphism?

(at least superficially, no).

What is the universal property of $U^{\hbar}(\mathfrak{g})$?

Let $\mathfrak{g}[1]$ be $\mathfrak{g}$ regarded as a graded algebra concentrated in degree 1, with a bracket of degree -1. Then every graded-Lie morphism ϕ of $\mathfrak{g}[1]$ into a graded $\mathbb{Q}[\hbar]$-algebra A factors uniquelly through $U^{\hbar}(\mathfrak{g})$:

(with $\deg \hbar = 1$)

$$\begin{tikzcd}
 & U^{\hbar}(\mathfrak{g}) \arrow[d, dashed, "\exists! \bar{\phi}"] \\
\mathfrak{g}[1] \arrow[ur, "i"] \arrow[r, "\phi"'] & A
\end{tikzcd}$$