



A. Suppose  $(\text{gr } Z^0) \circ \pi = \text{Id}$  Then  $\pi$  and  $\text{gr } Z^0$  are isomorphisms.

B. Identifying  $A = \text{gr } K$  and renaming  $Z^0$  to  $Z$ ,  $Z$  is an expansion for  $K$ .

Why was I so retarded back when I wrote the Tianjin/Hanoi handouts?

Precise Formulation Let  $K$  be filtered.

Assume  $A$  is graded,  $\pi: A \rightarrow K$

is onto, and we have a filtered

$Z^0: K \rightarrow A$  st.  $\text{gr } Z^0 \circ \pi = \text{Id}$ .

[ $Z^0$  is an  $A$ -expansion]. Then

$\pi: A \xrightarrow{\sim} \text{gr } K$  and  $Z = \pi \circ Z^0$  is an

expansion.