```
Star[a_, b_] := (a + b - 2 a b)/(1 - a b)
```

```
d = {Star[Star[a, b], c], Star[a, Star[b, c]]}
```

$$\left\{\frac{\frac{a+b-2ab}{1-ab}+c-\frac{2(a+b-2ab)c}{1-ab}}{1-\frac{(a+b-2ab)c}{1-ab}},\ \frac{a+\frac{b+c-2bc}{1-bc}-\frac{2a(b+c-2bc)}{1-bc}}{1-\frac{a(b+c-2bc)}{1-bc}}\right\}$$

```
d // Simplify
```

$$\left\{\frac{a+b-2ab+c-2ac-2bc+3abc}{1-ab-ac-bc+2abc},\ \frac{a+b-2ab+c-2ac-2bc+3abc}{1-ab-ac-bc+2abc}\right\}$$

```
{1, -1}.d // Simplify
```

```
0
```

```
Star[1, a]
```

```
1
```

```
Simplify[D[Star[f[x], f[y]], y]]
```

$$\frac{(-1+f[x])^2 f'[y]}{(-1+f[x]\,f[y])^2}$$

```
eqn = f'[x] == Simplify[
   D[Star[f[x], f[y]], y] /. {f[y] → 0, f'[y] → c}
  ]
```

$$f'[x] == c\,(-1+f[x])^2$$

```
DSolve[eqn, f[x], x]
```

$$\left\{\left\{f[x] \to \frac{-1+c\,x+C[1]}{c\,x+C[1]}\right\}\right\}$$

```
g[x_, c1_, c2_] := (x c1 + c2 - 1) / (c1 x + c2)
```

```
Simplify[
 g[x + y] == Star[g[x], g[y]]
]
```

$$\frac{1}{(-1+x+y)\,(x+y)} == 0$$

```
h[x_] = x / (x + 1)
```

$$\frac{x}{1+x}$$

```
Simplify[
 h[x + y]
]
```

$$\frac{x+y}{1+x+y}$$