Thomas: "Dual" Garside structures for finite-type Artin groups via quiver representations

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Coxeter groups & Artin groups:

Fix $n > 0$; $m_{ij} \in \{2, 3, \dots, \infty\}$ $1 \le i < j \le n$

$$\left\langle s_i : \begin{array}{l} s_i^2 = 1 \\ \underbrace{s_i s_j s_i \dots s_{j\text{ or }i}}_{m_{ij}} = \underbrace{s_j s_i s_j \dots}_{m_{ij}} \end{array} \right\rangle =: W$$

$A :=$ Same, but without $s_i^2 = 1$ $\quad \pi : A \to W$

__Example__ $m_{i,i+1} = 3$, $m_{ij} = 2$ otherwise gives $W =$ symmetric group S_{n+1}

$A =$ Braid group. B_{n+1}

If $|W| < \infty$ then W acts on $\mathbb{R}^n$ by reflections in linear subspaces.

The affine case: W acts on $\mathbb{R}^{n-1}$ by reflection about affine subspaces.

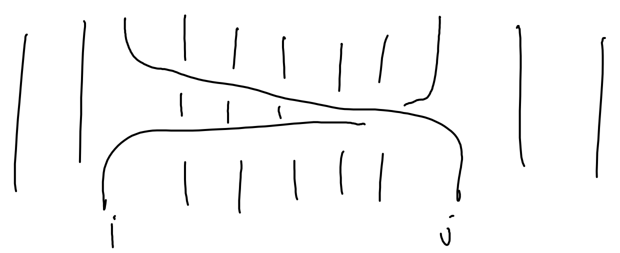
W is "simply-laced" if $m_{ij} \in \{2, 3, \infty\}$

"right angled" if $m_{ij} \in \{2, \infty\}$

"free" if all $m_{ij} = \infty$

The Birman-Ko-Lee presentation of $\text{Artin}(A_n) =: A$

$$\tau_{ij} = \sigma_i \sigma_{i+1} \dots \sigma_{j-1} \sigma_{j-2}^{-1} \sigma_{j-3}^{-1} \dots \sigma_i^{-1}$$

$=$  $\quad i < j$

<u>Relations</u> $\tau_{ik}\tau_{ij} = \tau_{jk}\tau_{ik} = \tau_{ij}\tau_{jk}$ for $i<j<k$

<u>Thm</u> $A = \langle \tau_{ij} \rangle /$ these rels.

Similar presentations exists if $|W|<\infty$ (Bessis)

$\tilde{A}_n$ (Digne)

free (Bessis)

<u>Today:</u> A uniform proof of a similar presentation in the finite, affine and right angled cases. (other than the non-crystallographic finite reflection groups)

The presentation: (W is simply laced)

Let Q be the quiver of $\{1, \dots n\}$,

if $m_{ij}=3$, put an arrow $i \bullet \!\to\! \bullet j$

$m_{ij}=\infty$ put many arrows $i \bullet \!\rightrightarrows\! \bullet j$

A representation X of Q is "exceptional" 4:40 if it is indecomposable and $Ext^1(X,X)=0$.

$E_1 \dots E_r$ is an exceptional sequence if E_i are exceptional & $Ext^1(E_j, E_i) = Hom(E_j, E_i) = 0$

There is a presentation D of the Artin group, whose generators are "exceptional quivers" and whose relations come from "mutations of exceptional sequences".