Rozansky@Colloq: Quantum field theories and their categories of boundary conditions

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4:07 PM

joint w/
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$$\begin{tikzcd}
\text{Topological QM} \arrow[rr] \arrow[dr] \arrow[dd] & & \text{A-model} \arrow[dd] & \\
& \text{B-model} \arrow[rr] \arrow[dd] & & \text{3d } \sigma\text{-model} \arrow[dd] \\
\begin{array}{c}\text{Diff geom/top}\\ \text{De-Rham corres}\end{array} \arrow[rr] \arrow[dr] & & \begin{array}{c}\text{Sympl geom/top}\\ \text{Fukaya}(X)\end{array} & \\
& \begin{array}{c}\text{Alg geom}\\ D^b(X)\end{array} \arrow[rr] & & \begin{array}{c}\text{Symp alg. geom.}\\ L(X)\end{array}
\end{tikzcd}$$

TQFT as a functor:

$\mathcal{C}ob_N$: objects: closed oriented $(N-1)$-manifolds

Morphisms: Cobordisms.

$\mathbb{C}$-Vect: Vector spaces over $\mathbb{C}$ (perhaps graded)

A TQFT is a functor

$$\mathcal{C}ob_N \to \mathbb{C}\text{-Vect}.$$

Example: $N=2$: $S^1 \longmapsto \widehat{S^1}$ (a v.s.)

$$\underbrace{\circ \cdots \circ}_{n} \longmapsto (\widehat{S^1})^{\otimes n}$$

 $\longmapsto m: (\widehat{S^1})^{\otimes 2} \to \widehat{S^1}$

... So $\widehat{S^1}$ is a Frobenius algebra.

σ-model: Σ - cobordism space time

X - target space

$$\begin{array}{c} \mathrm{maps}(\Sigma \longrightarrow X) \\ \Big\downarrow{\scriptstyle \mathrm{restr}} \\ \mathrm{maps}(\partial\Sigma \longrightarrow X) \end{array}$$

physics exercise: find the pushforward measure.

We imagine d (the de-Rham differential) to be a vector field with $dx^i = \eta^i$ where the η^i are odd variables in TX.

$\varphi \in \mathrm{Map}(\Sigma \to X)$ $\quad X$ - complex

$\Gamma(\varphi^*(\overline{T}X))$, replacing d by $\bar\partial$

$\rightsquigarrow \widetilde{\mathrm{maps}}(\Sigma \to X)$ (maps along with sections)

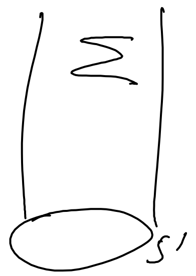
Q replaces $\bar\partial$: $\quad Q\varphi = 0 \quad Q\bar\varphi = \eta$

The measure: (g - a Kähler metric on X)

$$e^{i/\hbar S(\varphi, \bar\varphi, \eta, \theta, \rho)}$$

$$\theta \in \Gamma(\varphi^*(T^*X))$$

$$\rho \in \Gamma(\varphi^*(TX) \otimes T^*\Sigma)$$



$$S = \int_\Sigma g(d\varphi \wedge * d\bar\varphi) + g(\eta \wedge * d\rho) + \theta \wedge d\rho$$

$$\rightsquigarrow (S^1)^\wedge = \Omega^{0,\cdot}(\Lambda^\cdot \underset{\smile}{T}X), \bar\partial$$

$H^{\bullet}_{\bar\partial}(\Lambda^{\bullet} TX)$

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