Rozansky: A 2-category related to a holomorphic symplectic manifold

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joint w/
A. Kapustin.

Construct a 2-category $\ddot{L}(X,\omega)$,

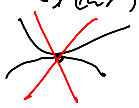
X complex, $\omega\in\Omega^{2,0}(X)$, $d\omega=0$

ω is nowhere degenerate.

Simplest objects: $Y\subset X$ Y is Lagrangian
(and hence holomorphic)

$\mathrm{Hom}(Y_1,Y_2)$ "=" $D^b(Y_1\cap Y_2)$

true if the intersection is "clean"

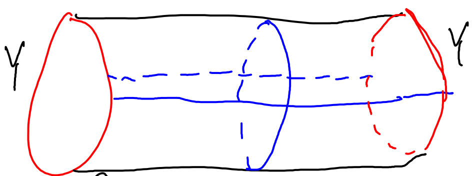


composition:

$$\begin{array}{ccc} & Y_1\cap Y_2\cap Y_3 & \\ \swarrow & \downarrow & \searrow^{\iota_{13}} \\ Y_1\cap Y_2 & Y_2\cap Y_3 & Y_1\cap Y_3 \\ \mathcal{E}_{12} & \mathcal{E}_{23} & \end{array}$$

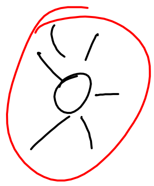
$$\mathcal{E}_{23}\circ\mathcal{E}_{13} = \iota_{13,*}\left(\iota_{12}^*\mathcal{E}_{12}\otimes\iota_{23}^*\mathcal{E}_{23}\right)$$

Cardy Condition:



$\rightsquigarrow V$

can be cut along a circle to get two semi-cylinders



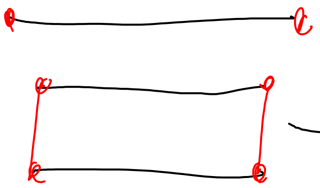
$\rightsquigarrow \mathcal{O}_Y\in D^b(X)$

Can be cut along two segments getting two rectangles.

$V=\mathrm{Ext}_X(Y,Y)$
$= H^\bullet_{\bar\partial}(\Lambda^\bullet N_{X/Y}) = \mathbb{K}2$

$\to D^b(Y)$

$S\mathbb{K}$



$\longrightarrow D^b(Y)$

$\longrightarrow \Delta_Y \in D^b(Y \times Y)$

S^*

So $V \cong \mathrm{Ext}_{Y\times Y}(\Delta_Y, \Delta_Y) = H^{\bullet}_{\bar\partial}(\Lambda^{\bullet} TY)$

So we better have the isomorphism *.

If Y is Lagrangian then $N_{X/Y} = T^*Y$

so $*2 = H^{\bullet}_{\bar\partial}(\Lambda^{\bullet} T^*Y)$ 4:28