Bezrukavnikov@Colloq: Characters of finite Chevalley groups and categorification

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4:11 PM

w/ Finkelberg, Ostrick.

G: Finite group

A representation $\rho : G \to GL(n, \mathbb{C})$ (homomorphism)

assume ρ is irreducible.

The character $\chi_\rho : G \to \mathbb{C}$: $\chi_\rho(g) = Tr(\rho(g))$

Questions 1. Describe $Irr(G)$

2. Compute all χ_ρ's.

Concentrate on G like $GL_n(\mathbb{F}_q)$; in general, $G = \underline{G}(\mathbb{F}_q)$ where $\underline{G}$ is an algebraic group over $\mathbb{F}_q$.

(Solved by Lusztig, sometimes w/ brute force)

Goal A more conceptual approach to this theory.

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characters are functions $(\underline{G} = X)(\mathbb{F}_q) \to \mathbb{C}$

Grothendieck's idea: such can come from:

$$\begin{tikzcd} X(\mathbb{F}_q) \arrow[d, leftrightarrow] & = & X(\overline{\mathbb{F}}_q)^{Frob} \arrow[d, leftrightarrow] \\ M^F & & \text{a manifold } M \text{ w/ an automorphism } F \end{tikzcd}$$

a function could come from an equivariant sheaf of f.d. v.s. on M: such a sheaf is a v.s. $\mathcal{F}_x$ $\forall x \in M$ + an isomorphism $\mathcal{F}_x \cong \mathcal{F}_{Fx}$

$\Rightarrow$ Get traces at the fixed points of F.

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Really, we will replace "sheafs" with "complexes of sheafs"; More specifically "perverse sheafs".

Induced representations: Given a representation ρ of $H < G$: $\quad (\rho: H \to GL(V))$

$$\mathrm{Ind}_H^G(\rho) = \{ f: G \to V : f(gh) = \rho(h^{-1}) f(g) \}$$

Suppose $G \supset \underset{\text{"Borel"}}{B} \longrightarrow \underset{\substack{\text{"cartan"} \\ \text{(Abelian)}}}{T}$

If $\chi \in T^\vee$, set $i_\chi := \mathrm{Ind}_B^G(\chi)$

For most χ this is irreducible, though

Consider $\chi = 1 \Rightarrow i_\chi = \mathbb{C}[G/B]$

Set $PS_1 := \{ \rho \mid \rho \subset i_1 \} \Leftrightarrow \rho^B \neq 0$ 4:49