2f(2x)

March-31-09
9:55 AM

$V: f(x) \longmapsto \sqrt{2} f(2x)$ is unitary:

classical reasoning

$$\|Vf\|^2 = \int (\sqrt{2} f(2x))^2 =$$

$$= \int 2 f^2(2x) = \int f^2(x) = \|f\|^2$$

Perturbative reasoning

$$V = \sqrt{2} \sum_{n=0}^{\infty} \frac{x^n \partial^n}{n!} \qquad V_0 = \sqrt{2}$$

$$V^* = \sqrt{2} \sum_{n=0}^{\infty} \frac{(-\partial)^n x^n}{n!} =$$

$$= \sqrt{2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \sum_{k=0}^{n} (n-k)! \binom{n}{k}^2 x^k \partial^k$$

$$= \sqrt{2} \sum_{k=0}^{\infty} \left(\sum_{n=k}^{\infty} \frac{(-1)^n}{n!} (n-k)! \binom{n}{k}^2 \right) x^k \partial^k$$

$$= \text{Divergent!} \qquad V_0^* = \sqrt{2} \sum_{n=0}^{\infty} (-1)^n$$

$V: f(x) \longmapsto \sqrt{1+2x}\, f(x+x^2)$ is unitary.

$$V_0 = \sqrt{1+2x}$$

$$V = \sqrt{1+2x} \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \partial^n$$

$$V^* = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \partial^n \left[x^{2n} \cdot \sqrt{1+2x} \right]$$

$$V_0^* =$$

Is there a general statement?