Pensieve Header: An improved MVA program with Jana Archibald.

```
<< KnotTheory`
```

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Loading KnotTheory` version of January 20, 2009, 17:44:36.984.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
MultivariableAlexander2[PD[Loop[_]]] := (1 / (#[1] - 1)) &
MultivariableAlexander2[K_] /; Head[K] =!= PD := MultivariableAlexander2[PD[K]]

MultivariableAlexander2[pd_PD] := MultivariableAlexander2[pd] = Module[
   {l, mat, skel, pd1, G, t, arcs, path, i, j, k, M, emb, done, pd2, rot, place},
   l = Length[pd];
   mat = Table[0, {2 * l}, {2 * l}];
   skel = Skeleton[pd];
   pd1 = List @@ pd;
   G = Table[0, {2 * l}, {l}];
   pd1 //. X[a_, b_, c_, d_] :> If[d == b + 1 || b - d > 1,
      {mat[[c, a]] = -t[b]; mat[[c, b]] = t[a] - 1;  mat[[c, c]] = 1},
      {mat[[c, a]] = -1; mat[[c, b]] = 1 - t[a] ; mat[[c, c]] = t[b]}
     ];
   arcs = Times @@ pd /. {
       X[i_, j_, k_, l_] /; (l - j == 1 || j - l > 1) :> path[k] path[i] path[j, l],
       X[i_, j_, k_, l_] /; (j - l == 1 || l - j > 1) :> path[k] path[i] path[l, j],
       P[i_, j_] :> path[i, j]
      } //. {
      path[a__, i_] path[i_, b__] :> path[a, i, b],
      path[a__, i_] path[b__, i_] :> Join[path[a, i], Reverse[path[b]]],
      path[i_, a__] path[i_, b__] :> Join[Reverse[path[b]], path[i, a]],
      path[a__, i_] path[i_] :> path[a, i],
      path[i_, a__] path[i_] :> path[a, i],
      path[i_] path[i_] :> path[i]
     };
   If[Length[arcs] === l, For[i = 1, i ≤ 2 * l, i++,
     G = ReplacePart[G, 1, {i, First[First[Position[arcs, i]]]}]
    ]];
   mat = mat /. t[a_] :> t[Position[skel, a][[1, 1]]];
   If[Length[arcs] === l,
    M = Factor[Simplify[
       Det[
         Delete[
          Transpose[Delete[
            Transpose[G].mat.G,
            Position[arcs, pd1[[1, 3]]][[1, 1]]
           ]],
```

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          Position[arcs, pd1[[1, 3]]][[1, 1]]
         ]
        ] / (t[Position[skel, pd1[[1, 3]]][[1, 1]]] - 1)
      ]],
    M = 0];
   emb = Table[Null, {Length[pd]}];
   done = Table[Null, {2 * Length[pd]}];
   emb[[1]] = 0;
   pd2 = pd;
   rot = Table[0, {Length[skel]}];
   place[i_, a_] := Module[
     {ni, na, arc, dir, oparc},
     arc = pd2[[i, a]];
     {{ni, na}} = Complement[Position[pd2, arc], {{i, a}}];
     If[emb[[ni]] === Null,
      emb[[ni]] = 3 - a + emb[[i]];
      pd2[[ni]] = RotateLeft[pd1[[ni]], na - 1];
      place[ni, #] & /@ {2, 3, 4},
      (* Else *) oparc = RotateLeft[pd2[[i]], 2][[a]];
      If[done[[arc]] === Null,
       done[[arc]] = 1;
       dir = If[arc - oparc == 1 || arc - oparc < -1, 1, -1];
       rot[[Position[skel, arc][[1, 1]]]] += dir * (emb[[ni]] - emb[[i]] + a - na - 2)
      ]
     ]
    ];
   place[1, #] & /@ {1, 2, 3, 4};
   k = -rot / 4;
   For[j = 1, j ≤ l, j++,
    k = ReplacePart[k,
      -1 + k[[Position[skel, pd[[j, 2]]][[1, 1]]]], Position[skel, pd[[j, 2]]][[1, 1]]]
   ];
   For[i = 1, i ≤ Length[k], i++,
    M *= t[i]^((1 / 2) * k[[i]])
   ];
   If[pd[[1, 4]] == pd[[1, 2]] + 1 || pd[[1, 2]] - pd[[1, 4]] > 1,
    M *= t[Position[skel, pd[[1, 1]]][[1, 1]]] * t[Position[skel, pd[[1, 2]]][[1, 1]]],
    M *= t[Position[skel, pd[[1, 1]]][[1, 1]]]
   ];
   Evaluate[M /. t → #] &
  ]

MV = MultivariableAlexander; MV2 = MultivariableAlexander2

MultivariableAlexander2

test1[L_] := (
   mv = MV[L][t]; mv2 = First[MV2[L][t]];
   Or @@ Map[
     (mv1 = mv /. t[i_] :> t[#[[i]]]; Head[Expand[Simplify[mv2 / mv1]]] =!= Plus) &,
     Permutations[Range[Length[Skeleton[L]]]]
    ]
  );
Print[# -> test1[#]] & /@ AllLinks[9];
```

KnotTheory::loading : Loading precomputed data in MultivariableAlexander4Links`.

KnotTheory::loading : Loading precomputed data in PD4Links`.

```
Link[9, Alternating, 1] → True
Link[9, Alternating, 2] → True
Link[9, Alternating, 3] → True
Link[9, Alternating, 4] → True
Link[9, Alternating, 5] → True
Link[9, Alternating, 6] → True
Link[9, Alternating, 7] → True
Link[9, Alternating, 8] → True
Link[9, Alternating, 9] → True
Link[9, Alternating, 10] → True
Link[9, Alternating, 11] → True
Link[9, Alternating, 12] → True
Link[9, Alternating, 13] → True
Link[9, Alternating, 14] → True
Link[9, Alternating, 15] → True
Link[9, Alternating, 16] → True
Link[9, Alternating, 17] → True
Link[9, Alternating, 18] → True
Link[9, Alternating, 19] → True
Link[9, Alternating, 20] → True
Link[9, Alternating, 21] → True
Link[9, Alternating, 22] → True
Link[9, Alternating, 23] → True
Link[9, Alternating, 24] → True
Link[9, Alternating, 25] → True
Link[9, Alternating, 26] → True
Link[9, Alternating, 27] → True
Link[9, Alternating, 28] → True
Link[9, Alternating, 29] → True
Link[9, Alternating, 30] → True
Link[9, Alternating, 31] → True
Link[9, Alternating, 32] → True
Link[9, Alternating, 33] → True
```

```
Link[9, Alternating, 34] → True
Link[9, Alternating, 35] → True
Link[9, Alternating, 36] → True
Link[9, Alternating, 37] → True
Link[9, Alternating, 38] → True
Link[9, Alternating, 39] → True
Link[9, Alternating, 40] → True
Link[9, Alternating, 41] → True
Link[9, Alternating, 42] → True
Link[9, Alternating, 43] → True
Link[9, Alternating, 44] → True
Link[9, Alternating, 45] → True
Link[9, Alternating, 46] → True
Link[9, Alternating, 47] → True
Link[9, Alternating, 48] → True
Link[9, Alternating, 49] → True
Link[9, Alternating, 50] → True
Link[9, Alternating, 51] → True
Link[9, Alternating, 52] → True
Link[9, Alternating, 53] → True
Link[9, Alternating, 54] → True
Link[9, Alternating, 55] → True
Link[9, NonAlternating, 1] → True
Link[9, NonAlternating, 2] → True
Link[9, NonAlternating, 3] → True
Link[9, NonAlternating, 4] → True
Link[9, NonAlternating, 5] → True
Link[9, NonAlternating, 6] → True
Link[9, NonAlternating, 7] → True
Link[9, NonAlternating, 8] → True
Link[9, NonAlternating, 9] → True
Link[9, NonAlternating, 10] → True
Link[9, NonAlternating, 11] → True
Link[9, NonAlternating, 12] → True
Link[9, NonAlternating, 13] → True
```

```
Link[9, NonAlternating, 14] → True
Link[9, NonAlternating, 15] → True
Link[9, NonAlternating, 16] → True
Link[9, NonAlternating, 17] → True
Link[9, NonAlternating, 18] → True
Link[9, NonAlternating, 19] → True
Link[9, NonAlternating, 20] → True
Link[9, NonAlternating, 21] → True
Link[9, NonAlternating, 22] → True
Link[9, NonAlternating, 23] → True
Link[9, NonAlternating, 24] → True
Link[9, NonAlternating, 25] → True
Link[9, NonAlternating, 26] → True
```

First::normal : Nonatomic expression expected at position 1 in First[0]. ≫

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

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General::stop : Further output of Power::infy will be suppressed during this calculation. ≫

```
Link[9, NonAlternating, 27] → True
Link[9, NonAlternating, 28] → True
```

```
{MV[Link[9, NonAlternating, 27]][t], MV2[Link[9, NonAlternating, 27]][t]}
```

```
{0, 0}
```

```
Flip[X[i_, j_, k_, l_]] := If[l == j + 1 || j - l > 1, X[j, k, l, i], X[l, i, j, k]];
VCube[pd_, l_List] := Module[
  {f},
  Expand[pd * Times @@ ((1 - f[#]) & /@ l)] //. pd1_PD * f[i_] :> MapAt[Flip, pd1, i]
 ]
```

```
Series[VCube[PD[#], {1, 2, 7}] /. pd_PD :> Jones[pd][E^x], {x, 0, 3}] & /@ AllLinks[8]
```

$$\{-9 x^3 + 0[x]^4,\ -12 x^3 + 0[x]^4,\ 12 x^3 + 0[x]^4,\ -12 x^3 + 0[x]^4,\ 12 x^3 + 0[x]^4,\ 12 x^3 + 0[x]^4,$$
$$3 x^3 + 0[x]^4,\ -15 x^3 + 0[x]^4,\ 0[x]^4,\ 12 x^3 + 0[x]^4,\ 15 x^3 + 0[x]^4,\ 12 x^3 + 0[x]^4,$$
$$12 x^3 + 0[x]^4,\ 15 x^3 + 0[x]^4,\ -18 x^3 + 0[x]^4,\ -18 x^3 + 0[x]^4,\ -18 x^3 + 0[x]^4,\ 18 x^3 + 0[x]^4,$$
$$18 x^3 + 0[x]^4,\ -24 x^3 + 0[x]^4,\ 36 x^3 + 0[x]^4,\ 9 x^3 + 0[x]^4,\ 9 x^3 + 0[x]^4,\ -18 x^3 + 0[x]^4,$$
$$-18 x^3 + 0[x]^4,\ -18 x^3 + 0[x]^4,\ -24 x^3 + 0[x]^4,\ -36 x^3 + 0[x]^4,\ -36 x^3 + 0[x]^4\}$$

```
Print[# → Series[VCube[PD[#], {1, 2, 7}] /. pd_PD :> MV2[pd][t] /. t[i_] → E^(h x[i]),
      {h, 0, 2}]] & /@ AllLinks[8];
```

$$\text{Link}[8, \text{Alternating}, 1] \to -x[2]^2 h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 2] \to \frac{1}{4}\left(x[1]^2 - 2\,x[1]\,x[2] + x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 3] \to \frac{1}{4}\left(-x[1]^2 + 2\,x[1]\,x[2] - x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 4] \to \frac{1}{4}\left(x[1]^2 - 2\,x[1]\,x[2] + x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 5] \to \frac{1}{4}\left(-x[1]^2 + 2\,x[1]\,x[2] - x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 6] \to \frac{1}{4}\left(-x[1]^2 + 2\,x[1]\,x[2] - x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 7] \to -x[1]\,x[2]\,h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 8] \to \frac{1}{4}\left(x[1]^2 + 2\,x[1]\,x[2] + x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 9] \to O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 10] \to \frac{1}{4}\left(-x[1]^2 + 2\,x[1]\,x[2] - x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 11] \to \frac{1}{4}\left(-x[1]^2 - 2\,x[1]\,x[2] - x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 12] \to \frac{1}{4}\left(-x[1]^2 + 2\,x[1]\,x[2] - x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 13] \to \frac{1}{4}\left(-x[1]^2 + 2\,x[1]\,x[2] - x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 14] \to \frac{1}{4}\left(-x[1]^2 - 2\,x[1]\,x[2] - x[2]^2\right) h^2 + O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 15] \to O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 16] \to O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 17] \to O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 18] \to O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 19] \to O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 20] \to O[h]^3$$

$$\text{Link}[8, \text{Alternating}, 21] \to O[h]^3$$

$$\text{Link}[8, \text{NonAlternating}, 1] \to x[2]^2 h^2 + O[h]^3$$

$$\text{Link}[8, \text{NonAlternating}, 2] \to x[2]^2 h^2 + O[h]^3$$

$$\text{Link}[8, \text{NonAlternating}, 3] \to O[h]^3$$

$$\text{Link}[8, \text{NonAlternating}, 4] \to O[h]^3$$

$$\text{Link}[8, \text{NonAlternating}, 5] \to O[h]^3$$

```
Link[8, NonAlternating, 6] → O[h]^3

Link[8, NonAlternating, 7] → O[h]^3

Link[8, NonAlternating, 8] → O[h]^3
```

```
Print[# → Series[VCube[PD[#], {1, 2, 7}] /. pd_PD :> MV2[pd][t] /. t[i_] → E^(h x[i]),
       {h, 0, 2}]] & /@ AllLinks[9];
```

```
Print[# → Series[VCube[PD[#], {1, 2, 7, 8}] /. pd_PD :> MV2[pd][t] /. t[i_] → E^(h x[i]),
       {h, 0, 2}]] & /@ AllLinks[9];
```

```
Print[# → Series[VCube[PD[#], {1, 2, 5, 7, 9}] /. pd_PD :> MV2[pd][t] /.
        t[i_] → E^(h x[i]), {h, 0, 3}]] & /@ AllLinks[11];
```