Very Large Numbers

January-14-09
2:47 PM

$It\,0 = 2$

$It\,1(n) = n+2$

$It\,2(f)(0) = 2;\ It\,2(f)(n+1) = f(It\,2(f)(n))$

$It\,3(F)(f)(0) = f(2);\ It\,3(F)(f)(1) = F(f)(2);\ It\,3(F)(f)(2) = F(F(f))(2)$ and so on

$It\,4(\mathbf{F})(F)(f)(0) = F(f)(2);\ It\,4(\mathbf{F})(F)(f)(1) = \mathbf{F}(F)(f)(2)$ and so on

and so on

$f(n) = It\,n(It\,n-1)\ldots(It\,1)(It\,0);\ f(67)$

$It_4(\mathbf{F})(F)(f)(n) = $
$= \mathbf{F}^n(F)(f)(2)$

$S_0 = \mathbb{N} \qquad It_0 \in S_0 \qquad It_0 = 2$

$S_1 = S_0^{S_0} = \mathbb{N}^{\mathbb{N}} \qquad It_1 \in S_1 \qquad It_1(n) = n+2$

$S_2 = S_1^{S_1}$

$It_0 = 2$

$It_1(n) = n+2$

$n > 1$

$It_n(F_1)(F_2)\ldots(F_{n-1})(m) = F_1^m(F_2)(F_3)\ldots(F_{n-1})(2)$

$f(2) = It_2(It_1)(2) = It_1(It_1(2)) = It_1(2+2) = 6$

$f(3) = It_3(It_2)(It_1)(2) = It_2(It_2(It_1))(2) \qquad 2/1^{2^2}$

$2/1^{2^3}$

$= (It_2(It_1) \circ It_2(It_1))(2) \qquad 2/1^2/^2$

$= It_2(It_1)(It_2(It_1)(2)) \qquad 2/1^2/^2$

$= It_2(It_1)(6) \qquad 6/1^2$

$= It_1^6(2) = 14 \qquad 2/111111$
$= 14$

$f(4) = It_4(It_3)(It_2)(It_1)(2) = \qquad 2/1^{2^{3^4}}$

$= (It_3 \circ It_3)(It_2)(It_1)(2) \qquad 2/1^{2^{33}}$

$= It_3(It_3(It_2))(It_1)(2) \qquad 2/1^{2^{33}}$

$= (It_3(It_2) \circ It_3(It_2))(It_1)(2) \qquad 2/1^{2^3 2^3}$

$= \mathrm{It}_3(\mathrm{It}_2)\left(\mathrm{It}_3(\mathrm{It}_2)(\mathrm{It}_1)\right)(2)$

$= (\mathrm{It}_2 \circ \mathrm{It}_2)\left(\mathrm{It}_3(\mathrm{It}_2)(\mathrm{It}_1)\right)(2) \qquad 2|\,|^{2^3 2 2}$

$= \mathrm{It}_2\left(\mathrm{It}_2\left(\mathrm{It}_3(\mathrm{It}_2)(\mathrm{It}_1)\right)\right)(2)$

$= \mathrm{It}_2\left(\mathrm{It}_3(\mathrm{It}_2)(\mathrm{It}_1)\right)\left(\mathrm{It}_2\left(\mathrm{It}_3(\mathrm{It}_2)(\mathrm{It}_1)\right)(2)\right) \; 2|\,|^{2^3 2}|^{2^3 2}$

$= 2|\,|^{2^3}|^{2^3}|^{2^3 2}$

$= 14|\,|^{2^3}|^{2^3 2}$

$= 2|\,|^{22222222222222}|^{2^3 2}$

$= 2|\,|^{2222222222222}|^{2222222222222}|^{2^3 2}$

$= 2|\,|^{222222222222}|^{12\cdot 2}|^{13\cdot 2}|^{2^3 2}$

$= 2|\,|^{11\cdot 2}|^{11\cdot 2}|^{12\cdot 2}|^{13\cdot 2}|^{2^3 2}$

$= 2|\,|^{2}|^{2\cdot 2}|^{3\cdot 2} \dots |^{13\cdot 2}|^{2^3 2}$

$= 2|\,|\,|\,|^{22}|^{3\cdot 2} \dots |^{13\cdot 2}|^{2^3 2}$

$= 6|\,|^{22}|^{3\cdot 2} \dots |^{13\cdot 2}|^{2^3 2}$

$= 2|\,|^2|^2|^2|^2|^2|^2|^{3\cdot 2} \dots |^{13 2}|^{2^3 2}$

$= 6|\,|^2|^2|^2|^2|^2|^{3\cdot 2} \dots |^{13\cdot 2}|^{2^3 2}$

$= 14|\,|^2|^2|^2|^2|^{3\cdot 2} \dots |^{13\cdot 2}|^{2^3 2}$

$= 30|\,|^2|^2|^2|^{3\cdot 2} \dots |^{3\cdot 2}$

$= 254|\,|^{222}|^{4\cdot 2} \dots |^{13\cdot 2}|^{2^3 2}$

$= 2|\,\underbrace{|^{22} \dots |^{22}}_{254}|^{4\cdot 2} \dots |^{13\cdot 2}|^{2^3 2}$

$= 2|\,|^2|^2\underbrace{|^{22} \quad |^{22}}_{253}|^{4\cdot 2} \dots |^{13\cdot 2}|^{2^3\cdot 2}$

$= 14|\,|^{22}\underbrace{|^{22} \quad |^{22}}_{252}|^{4\cdot 2} \dots |^{13\cdot 2}|^{2^3\cdot 2}$

$= 2|\,\underbrace{|^2|^2 \dots |^2}_{14}\underbrace{|^{22} \dots |^{22}}_{252}|^{4\cdot 2} \dots |^{13\cdot 2}|^{2^3\cdot 2}$

$$= 2^{14}-2 \mid \underbrace{1^{22} \quad 1^{22}}_{252} \; 1^{4\cdot 2} \ldots 1^{13\cdot 2} \; 1^{2^3\cdot 2}$$

$$= 2^{2^{14}}-2 \ldots$$

$$= \left.2^{2^{2^{2}}}\right\} 250 \mid 1^{2222} \; 1^{5\cdot 2} \ldots 1^{13\cdot 2} \; 1^{2^3\cdot 2}$$

$$= \ldots \quad \sim A(A(14))$$

See also ItaisIt.nb

Question: How does this compare with

Goodstein's theorem <http://en.wikipedia.org/wiki/Goodstein%27s_theorem>