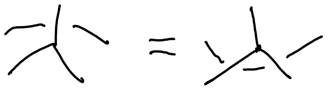
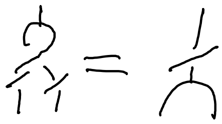
The Two F Equations

December-27-08
3:47 PM

$F^{-1}e(x+y)F = e(x)e(y)$

$F^{23}R^{1,23} = R^{12}R^{13}F^{23}$ $\longleftrightarrow$  Eq1

$RF^{21}e(-t) = F$ $\longleftrightarrow$  Eq2

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solving functional equations with $f(x,y)$ & $f(y,x)$:

$\Rightarrow$ switch to $u = x+y$ & $v = x-y$,

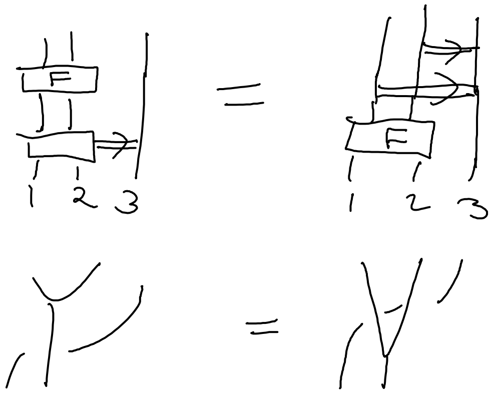
solve eqn's in $g(u,v)$ & $g(u,-v)$

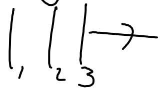
$g(u,v) = g_{even} + g_{odd}$

$g(u,-v) = g_{even} - g_{odd}$

Tail scattering for eq_1:

lhs_1 rhs_1

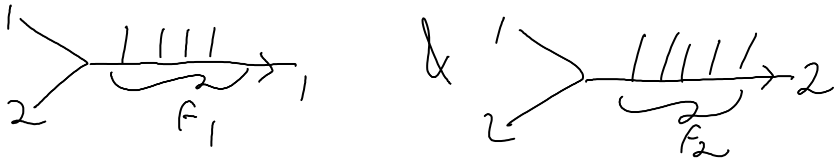


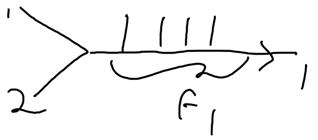
1. only interesting to scatter 

2. should have tail-degree 1 in x_3

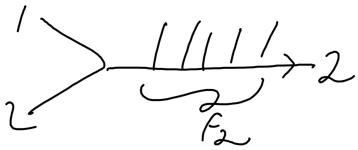
Linearization

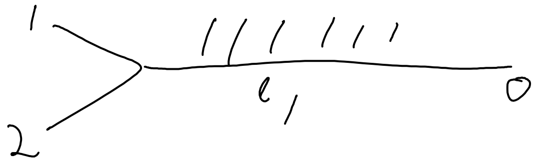
Beyond low degrees, F is determined by





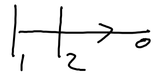
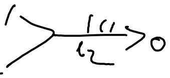
&



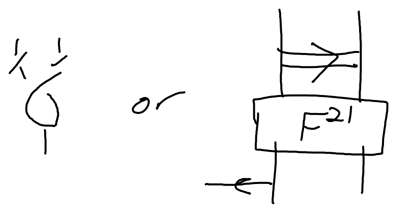
And eq_1 is determined by 

Q: What is the linearization of $\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \longmapsto e_1$?
(in lhs-rhs)

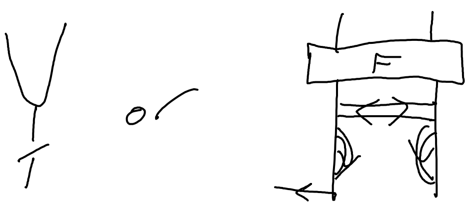
Ans: It is $x_3(x_1 f_1 + x_2 f_2)$ (check with experiment!)
on 090105

Q Likewise, what is the linearization of $\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \longmapsto e_2$?
(scatter , e_2 is the coefficient of  in lhs-rhs)

lhs2 = Ar[1, 0] // F21 // S[sigma[1, 2]] // S[Exp[1/2 Ar[1, 1]]] // S[Exp[1/2 Ar[2, 2]]]



rhs2 = Ar[1, 0] // S[Exp[Expand[1/2 (Ar[1, 1] + Ar[1, 2] + Ar[2, 1] + Ar[2, 2])]]] // F



Ans It is $+x_1\left(f_2(x_2, x_1) + f_1(x_1, x_2)\right)$
(checks with experiment!)
on 090105

For the purpose of deciding uniqueness:
what is the kernel of

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \longmapsto \begin{pmatrix} x_1 f_1 + x_2 f_2 \\ f_1(x_1, x_2) + f_2(x_2, x_1) \end{pmatrix} \ ?$$

First component: If $x_1 f_1 + x_2 f_2 = 0$ then for some g,
$f_1 = x_2 g$ & $f_2 = -x_1 g$

Second component:

Second component:

$$f_1(x_1, x_2) + f_2(x_2, x_1) = x_2 g(x_1, x_2) - x_2 g(x_2, x_1)$$
$$= x_2 (g - g^T)$$

$\Rightarrow g = g^T \Rightarrow$ g might be any function of $x_1 + x_2$ & $x_1 \cdot x_2$.

Is there a conceptual explanation for this?