Mod CC Cyclic Symmetry of the Vertex

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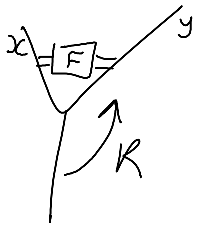
(This can come before anything involving υ & wheels)

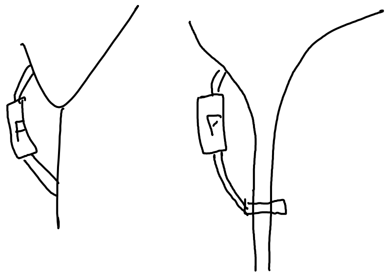
The vertex is

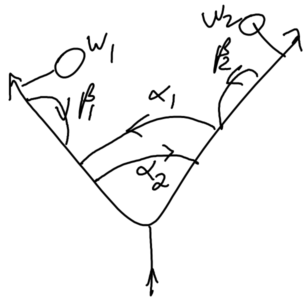
$$F = \exp\left(a_1 \,{}_x\,{}_y + a_2 \,{}_x\,{}_y + H(f_1)\,{}_x\,{}_y + H(f_2)\,{}_x\,{}_y\right)$$



Q: How does R, the 120° counterclockwise rotation, acts on F?

Some parts are scattering-invisible!?





The linear terms:

$$\begin{aligned}
R:\ \alpha_1 &\to -\alpha_2-\beta_1 \qquad \xrightarrow{\text{mod } W} \cancel{\alpha_1}\cancel{+\beta_1}\cancel{-\alpha_1}-\alpha_2-\cancel{\beta_1}-\beta_2 \longrightarrow \alpha_1+\beta_1-\beta_1=\alpha_1\\
\alpha_2 &\to -\alpha_1-\beta_1\\
\beta_1 &\to \alpha_1+\alpha_2+\beta_1+\beta_2+w_1+w_2\\
\beta_2 &\to \beta_1\\
w_1 &\to -w_1-w_2\\
w_2 &\to w_1
\end{aligned}$$

$$\left.\begin{aligned}
(1+R+R^2)\alpha_1 &= \alpha_1-\alpha_2-\beta_1-\alpha_2-\beta_2 = \alpha_1-\alpha_2-\beta_1-\beta_2\\
(1+R+R^2)\beta_1 &= \beta_1+\alpha_1+\alpha_2+\beta_1+\beta_2+\beta_2\\
&= 2\beta_1+2\beta_2+\alpha_1+\alpha_2
\end{aligned}\right\}\ \text{mod } W$$

$\Rightarrow$ $3\alpha_1-\alpha_2$ is also R symmetric.