we want to define a special type of categories that "look like" A-modules.

<u>Def</u> A category is "additive" if

* The initial & final object coincide & are called 0.
* $\forall X, Y$ $X \sqcup Y = X \sqcap Y$ exist & are equal.

  (Exercise: it follows from the axioms that $\sqcup$ & $\sqcap$ are associative (in the appropriate sense))

* $\mathrm{Hom}_A(X, Y)$ is always an Abelian group, the composition of morphisms is bilinear.

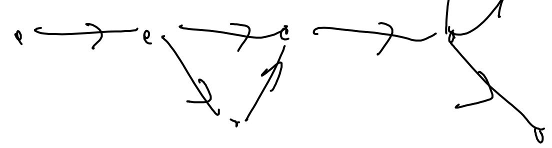
<u>Examples</u>: Vector spaces, A-modules, A-ring, Vector bundles on X, Topological Vector spaces, filtered vector spaces: $(V_i \subset V;\ i<j \Rightarrow V_i \subset V_j)$

<u>Comment</u> In filtered vector spaces,

$$Id : (V, \{0\}) \to (V, \{V_i\})$$

is "bijective" but not invertible.

<u>Def</u> A "quiver" is an oriented graph without oriented cycles:

Q : 

edges E
vertices V

A representation of Q is a functor from the category generated by Q to Vect.

"Rep(Q)" makes an additive category.

Topological <u>Example</u>: A : The stable homotopy category.

Topological Example: $\mathcal{A}$: The stable homotopy category.
(addition of morphisms in not at all obvious)

Example: If C is any category and $\mathcal{A}$ is an additive category, then $\mathrm{Fun}(C, A)$ is an additive category.

Example X: Topological space

$C^o(X)$: objects are open subsets
$U \to V$ if $U \supset V$

$\mathrm{Fun}(C^o(X), \mathcal{A})$ is "pre-sheaves on X with values in $\mathcal{A}$", aka $\text{Pre-Sh}(X)$

Example Example: $\mathcal{O}(U) = \left\{ \begin{matrix} \text{continuous functions} \\ \text{on } U \end{matrix} \right\}$

Def "Additive functor"

Homological Algebra in an additive Category

Def $\mathrm{Com}(\mathcal{A})$: objects $\cdot \xrightarrow{d} \cdot \xrightarrow{d} \cdot \xrightarrow{d} \cdot \xrightarrow{d} \cdot$ w/ $d^2=0$

morphisms: as expected.

$\mathrm{Com}(\mathcal{A})$ is an additive category.

Def Homotopy between morphisms of complexes.