# Degree 2 for Virtual Knots

January-13-09
1:24 PM

What are $\vec{\mathcal{A}}(\text{anything})/6T$?
Does every weight system integrate?
(for knots, for links, for long knots)
(for braids, for tangles)
What would x-knots be?
Is there a local UFTI?
Is there an extension to vKTGs?

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Screen clipping taken: 14/01/2009, 9:24 AM from AcademicPensieve/Projects/WKO/DegreeTwo.nb.

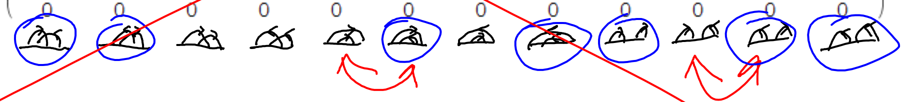
In[13]:= `ShowMatrix[RowReduce[mat]]`

Out[13]//MatrixForm=

outdated

| 1342 | 2431 | 1324 | 3142 | 1423 | 1432 | 2341 | 3241 | 1234 | 1243 | 2134 | 2143 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 2 | 0 | 0 | -1 | 0 | -2 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | -2 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | -1 | 0 | -1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | -1 | -1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

basis



Basis:

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Better yet:

Screen clipping taken: 14/01/2009, 9:39 AM from AcademicPensieve/Projects/WKO/DegreeTwo.nb.

In[12]:= `ShowMatrix[RowReduce[mat]]`

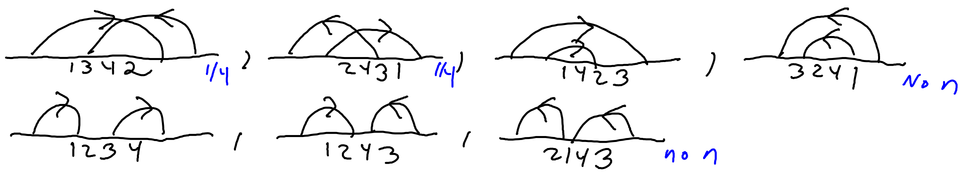
Out[12]//MatrixForm=

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1342 | 2431 | 1324 | 3142 | 1432 | 1423 | 2341 | 3241 | 1234 | 2134 | 1243 | 2143 |
| 0 | 0 | 1 | 0 | 0 | -2 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | -2 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | -1 | 0 | -1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | -1 | -1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1/4 | 1/4 | 1/4 | 1/4 | | | lin $n$ | No $n$ | | lin $n$ | lin $n$ | No $n$ |

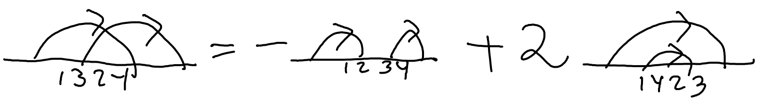
basis

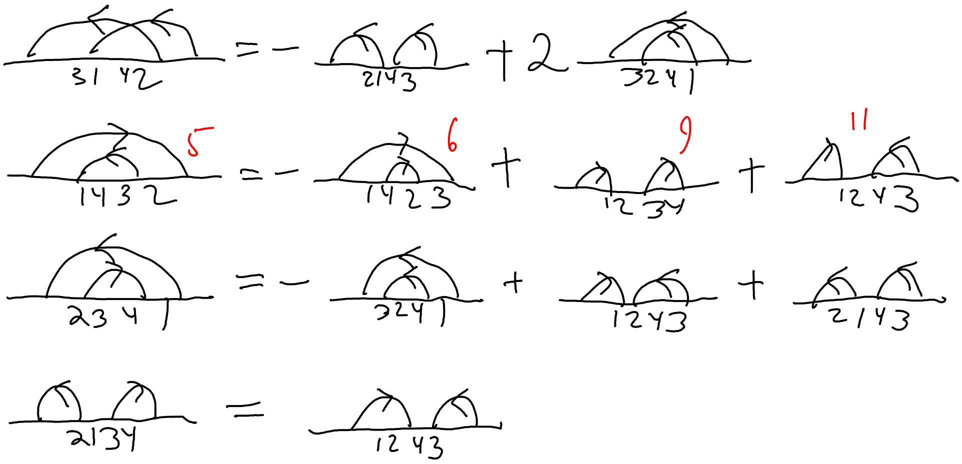
$gl(N)$ (defining)

Basis:




Reductions:




What does $gl(N)$ see? In the defining rep,

```
In[63]:= (# → Wgl[#][n, k]) & /@ diags
```

Out[63]= $\{$Diag[ar[1, 3], ar[4, 2]] $\to \frac{1}{4}$, Diag[ar[2, 4], ar[3, 1]] $\to \frac{1}{4}$,

Diag[ar[1, 3], ar[2, 4]] $\to \frac{1}{4}$, Diag[ar[3, 1], ar[4, 2]] $\to \frac{1}{4}$,

Diag[ar[1, 4], ar[3, 2]] $\to -\frac{1}{4}+\frac{k}{2}-\frac{k^2}{2}+\frac{n^2}{2}$,

Diag[ar[1, 4], ar[2, 3]] $\to \frac{1}{4}-\frac{k}{2}+\frac{k^2}{2}+\frac{n}{2}-k\,n+\frac{n^2}{2}$,

Diag[ar[2, 3], ar[4, 1]] $\to -\frac{1}{4}+\frac{k}{2}-\frac{k^2}{2}-\frac{n}{2}+k\,n$,

Diag[ar[3, 2], ar[4, 1]] $\to \frac{1}{4}-\frac{k}{2}+\frac{k^2}{2}$,

Diag[ar[1, 2], ar[3, 4]] $\to \frac{1}{4}-k+k^2+n-2\,k\,n+n^2$,

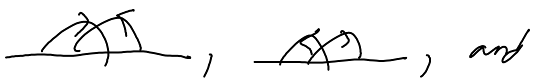
Diag[ar[2, 1], ar[3, 4]] $\to -\frac{1}{4}+k-k^2-\frac{n}{2}+k\,n$,

Diag[ar[1, 2], ar[4, 3]] $\to -\frac{1}{4}+k-k^2-\frac{n}{2}+k\,n$,

Diag[ar[2, 1], ar[4, 3]] $\to \frac{1}{4}-k+k^2\}$

Screen clipping taken: 14/01/2009, 10:08 AM

What if I also mod out by "short arrows commute with anything" (  )?

– All that's left is , and the three "framing numbers": 

⇒ Not quite "Tails Commute", for  and  remain distinct.