Costello@Colloq: Functional measures in perturbative quantum field theory

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A formal picture of QFT: Input:

$$S(\phi) = \int_M \mathcal{L}(\phi) \; ; \text{ typically } \mathcal{L}(\phi) = \int_M \phi(\Delta + m^2)\phi \text{ + perturbations.}$$

Output: The "statistical system" of a random field $\phi \in C^\infty(M, \mathbb{R})$ wrt to the measure

$$e^{iS(\phi)/\hbar} \, d\mu_{\text{Lebesgue}} \quad \text{or} \quad e^{-S(\phi)/\hbar} \, d\mu$$

That is, $O_f(\phi) := \int_M f\phi$; we want

$$\mathbb{E}(O_{f_1}, O_{f_2}, \ldots O_{f_n}) \text{ ; Typically a distribution on } M^n$$

Minlos' theorem says that the free Euclidean measure for $\phi(\Delta + m^2)\phi$ actually does exist, and is supported on distributions

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<u>Def</u> A formal measure on $\mathcal{D}(M)$ is a cont. linear map

$$\prod_{n \geq 0} \left(C^\infty(M^n)^{S_n} \right) \longrightarrow \mathbb{R}[[\hbar]]$$

<u>Thm</u> There is a bijection between

1. Lagrangians $\mathcal{L}(\phi) = \sum_{i \geq 0} \hbar^i \mathcal{L}_i(\phi)$
   where $\mathcal{L}_0(\phi) = \phi(\Delta + m^2)\phi$ + cubic & higher
   up to the addition of a total derivative

2. Formal measures satisfying a "locality" property: