

Motivation:

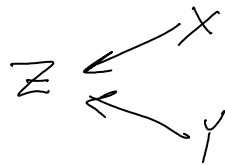
$$\text{Hom}_{\text{set}}(X \sqcup Y, Z) = \text{Hom}_{\text{set}}(X, Z) \times \text{Hom}_{\text{set}}(Y, Z)$$

$$X \sqcap Y := X \times Y$$

$$\text{Hom}_{\text{set}}(Z, X \sqcap Y) = \text{Hom}(Z, X) \times \text{Hom}(Z, Y)$$

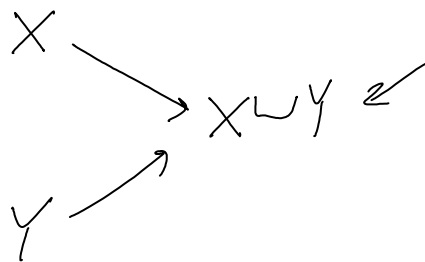
Category of Diagrams $\text{Diag}(X \sqcup Y)$ (given some X, Y in some base category)

objects:



morphisms: commutative diagrams between such.

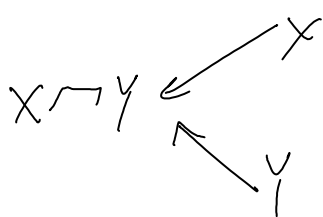
Def



the initial object in $\text{Diag}(X \sqcup Y)$

Likewise, $\text{Diag}(X \sqcap Y)$ has objects $\{Z \begin{matrix} \rightarrow X \\ \rightarrow Y \end{matrix}\}$ with obvious morphisms

Def



the final object in $\text{Diag}(X \sqcap Y)$

Example: Vect

Example Commutative rings with 1 Comm

$$A \sqcup B = A \otimes B \quad A \sqcap B = A \oplus B$$

agrees with the contravariant functor

$$\text{Fun}: \text{Sets} \rightarrow \text{Comm}$$

$$\text{by } S \longmapsto \text{Fun}(S) = \text{Functions on } S$$

$$\text{Indeed, } \text{Fun}(S \times T) = \text{Fun}(S) \otimes \text{Fun}(T)$$

Indeed, $\text{Fun}(S \times T) = \text{Fun}(S) \otimes \text{Fun}(T)$
 $\text{Fun}(S \cup T) = \text{Fun}(S) \oplus \text{Fun}(T)$

Example $C = \text{Rings}$ (not nec. commutative)

$$A \sqcap B = A \otimes B$$

$$A \sqcup B = \text{the free product of } A \text{ \& } B$$

Example $C = \text{Groups}$ (not nec. Abelian)

$$G \sqcap H = G \times H \quad (\text{direct product})$$

$$G \sqcup H = G * H \quad (\text{free product})$$

Motivation $C = A\text{-modules}$

$$\begin{array}{ccc|c}
 F: M \longrightarrow N & & & \text{Diag}(\ker(F)) := \text{The category} \\
 \ker(F) \xrightarrow{k} M & & & \text{with objects} \\
 \downarrow & & \downarrow F & \\
 0 \longrightarrow N & & & \\
 & & X \longrightarrow M & \\
 & & \downarrow & \downarrow F \quad (\text{obvious} \\
 & & 0 \longrightarrow N & \text{morphisms})
 \end{array}$$

$\ker(F)$ is a final object in $\text{Diag}(\ker(F))$

Def Let \mathcal{C} be a category with an object 0 which is both initial & final. Given $f: X \rightarrow Y$ in \mathcal{C} , $\ker(f)$ is a final object in the likewise-defined $\text{Diag}(\ker(f))$

Example For the category of pointed sets,
 $\ker(f) = f^{-1}(*)$

Motivation $F: M \rightarrow N$

$$\begin{array}{ccc} M & \xrightarrow{F} & N \\ \downarrow & & \downarrow \\ 0 & \rightarrow & \text{coker } F \end{array}$$

$\text{Diag}(\text{coker}(F)) =$

$$\left\{ \begin{array}{ccc} M & \xrightarrow{F} & N \\ \downarrow & & \downarrow \\ 0 & \rightarrow & Z \end{array} \right\} \quad \text{obvious morphisms}$$

Def Cokernels in general are the initial objects in $\text{Diag}(\text{Coker}(F))$

Example In pointed sets, $\text{coker}(F) = Y/A(x) \sim *$

Example $\mathcal{C} = \text{Top Vector spaces}$; there are morphisms w/ vanishing ker & coker, yet which aren't iso:

$$V^{\text{discrete}} \xrightarrow{\text{id}} V$$