KAL-090128: Lie bialgebras, gl(N), framing v-knots

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9:53 AM

Results with Louis:

n	0	1	2	3	4
$\dim \vec{A}_n$	1	2	7	27	139
$\dim \operatorname{im} T_{gl(N)}$	1	2	6	~ 22	less
$\dim \operatorname{im} T^{bialg}_{gl(N)}$	1	2	7	27	≥ 116

http://katlas.math.toronto.edu/drorbn/AcademicPensieve/Projects/Arrow_Diagrams_and_gl(N)/index.html

Something must be right and something is going on. Are there really two gl(N) constructions? Which of them is "right"?

[some random thoughts on the matter are below]

* Reminder: $\vec{A}/6T$

* Reminder: Lie bialgebras & arrow weight systems.

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$$\underset{\{e^L_{ij}\}_{i\geq j}}{L} \oplus \underset{\{e^U_{ij}\}_{i\leq j}}{U} = \underset{\{e_{ij}\}}{gl(N)} \oplus \underset{\{h_i = \frac{1}{2}(e^U_{ii} - e^L_{ii})\}}{H}$$

with $h_i := e_{ii} = \frac{1}{2}(e^U_{ii} + e^L_{ii})$

* 6T For gl(N)

(That is, some specific $r \in gl(N) \otimes gl(N)$ solves the IYBE [6T] in $U(gl(N))^{\otimes 3}$)

*

$$\begin{tikzcd}
\vec{A}(\uparrow) \arrow[r, "T_{gl(N)}"] \arrow[d, "d"] & U(gl(N)) \arrow[d, leftrightarrow] \\
\vec{A}(\uparrow\uparrow^{ab}) \arrow[r, "T_{gl(N);H}"] & U(gl(N) \oplus H) = U(gl(N)) \otimes S(H)
\end{tikzcd}$$

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* In Lie world, $\vec{\mathcal{A}}(\uparrow^{ab}\dots)$ represents the Cartan
* In topology, $\vec{\mathcal{A}}(\uparrow^{ab} X)$ represents a "homology class" in the complement of X.
* Example: $\mathcal{A}(\uparrow^{r} \uparrow^{ab}) \cong \mathcal{A}(\uparrow)$ as vector spaces, though I'm not sure how compatible this is with all the operations available on $\mathcal{A}(\uparrow)$
* Remember that even on framed v-knots, 6T is not the only relation.

This all suggests that it may be that the right objects to study are v-framed v-knots - these would be v-knots with a v-homology class in their complement.

[I don't expect this to be the truth, only a step in the right direction]