Tingley: Quiver Grassmannians and a Geometric Realization of the Schutzenberger Involution

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$\mathfrak{g}$: finite dim complex simple Lie algebra

$U_q(\mathfrak{g})$: QUEA, generated by E_i, F_i, K_i

Involution:

$$C_\xi: \quad E_i \longrightarrow F_{\theta(i)}, \quad F_i \longrightarrow E_{\theta(i)}, \quad K_i \longrightarrow K_{\theta(i)}^{-1}$$

θ is an involution of the Dynkin diagram:

C_ξ is an algebra automorphism and a co-algebra antiautomorphism.

There is:

$$\sigma^{br}: V_\lambda \otimes V_\mu \longrightarrow V_\mu \otimes V_\lambda$$

$$\xi_{V_\lambda}: V_\lambda \longmapsto V_\lambda^{low}$$

compatible with C_ξ.

$$X_{V_\lambda}: V_\lambda \longmapsto i^{\langle\lambda, 2\rho^\vee\rangle} q^{-} \, V_\lambda^{low}$$

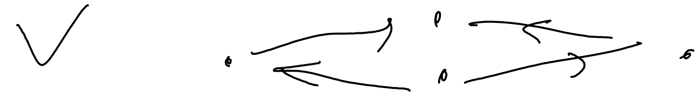
<u>Thm</u> (K-R, L-S) [Kir Resh, Soib] $\sigma^{br} = \mathrm{Flip} \circ (X^{-1} \otimes X^{-1}) \Delta X$

on each irreducible factor of $V_\lambda \otimes V_\mu$

A quiver is a directed graph:

$$Q: \quad \circ \underset{-}{\overset{+}{\rightleftarrows}} \circ \underset{-}{\overset{+\ \epsilon(e)}{\rightleftarrows}} \circ$$

a representation:



$$\mathbb{C} \rightleftarrows \mathbb{C}^2 \rightleftarrows \mathbb{C}$$

"moment map" $\mu : Rep(Q, V) \longrightarrow \prod End(V_i)$

by $\qquad x \longmapsto \prod_i \sum_{\epsilon(l)=i} \epsilon(l)\, x_{\bar{l}}\, x_l$

Require $\mu = 0$.

$$\Lambda_Q(V) = \quad Reps(Q, V) \text{ s.t. } \mu = 0$$

Too technical to follow.