Reading Furusho

December-01-08
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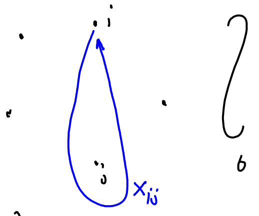
From http://front.math.ucdavis.edu/0702.5128

Here $\hat{\mathfrak{P}}_5$ stands for the completion (with respect to the natural grading) of the pure sphere braid Lie algebra $\mathfrak{P}_5$ [I] with 5 strings; the Lie algebra generated by X_{ij} ($1 \leqslant i, j \leqslant 5$) with clear relations $X_{ii} = 0$, $X_{ij} = X_{ji}$, $\sum_{j=1}^{5} X_{ij} = 0$ ($1 \leqslant i, j \leqslant 5$) and $[X_{ij}, X_{kl}] = 0$ if $\{i,j\} \cap \{k,l\} = \emptyset$. It is a quotient of $\mathfrak{a}_4$ (cf. §2).

[I] is Ihara



Between the Lie algebra $\mathfrak{a}_4$ in theorem 1 and $\hat{\mathfrak{P}}_5$ in theorem 3 there is a natural surjection $\tau : \mathfrak{a}_4 \to \hat{\mathfrak{P}}_5$ sending t_{ij} to X_{ij} ($1 \leqslant i, j \leqslant 4$). Its kernel is generated by $\Omega = \sum_{1 \leqslant i < i \leqslant 4} t_{ij}$. We also denote its induced morphism $U\mathfrak{a}_4 \to U\hat{\mathfrak{P}}_5$ by τ. On

($\mathfrak{a}_4$ is the usual projectivized pure braid group)

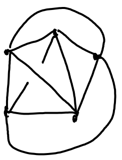
Does it generalize? What does it mean?

Eliminating X_{i5}: Let $\{i,j,k,l\} = \{1,2,3,4\}$

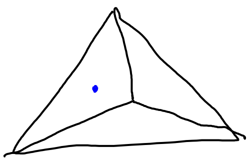
$[X_{ij}, X_{k5}] = 0$ becomes $0 = [X_{ij}, X_{ik} + X_{jk} + X_{lk}]$

$= [X_{ij}, X_{ik} + X_{jk}]$

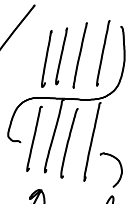
So 4T holds, though we've used 5-specific properties.



A planar K_5 with only one crossing.



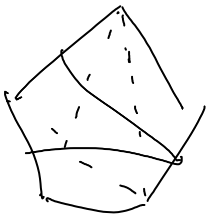
SB_5 (the group whose projectivization is the $\mathfrak{P}_5$ of above) is the group of spherical braids on 5 strands $\underbrace{\text{modulo the full twist.}}$

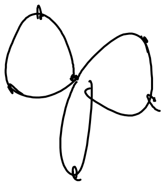
$SB_5 = B_5 /$  $= I = \Delta$

spherical

Garside's "full twist".

This follows from spheriallity.





$$\varphi(X_{12}, X_{23}) + \varphi(X_{34}, X_{45}) + \varphi(X_{51}, X_{12}) + \varphi(X_{23}, X_{34}) + \varphi(X_{45}, X_{51}) = 0$$

$$\varphi(t_{12}, t_{23}) + \varphi(t_{34}, -t_{41}-t_{42}-t_{43})$$

Proposition 3.1. *The quadratic dual $\Lambda_n^!$ of Λ_n is the algebra U_n generated over $\mathbb{Z}$ by μ_{ijk}, $1 \le i,j,k \le n$, which are antisymmetric in ijk, with defining relations*

$$\sum_i \mu_{ijk} = 0, \ \ [\mu_{ijk}, \mu_{pqr}] = 0$$

for distinct i,j,k,p,q,r (with the obvious action of S_n). It is also generated by μ_{ijk} with $1 \le i,j,k \le n-1$ with defining relations

$$[\mu_{ijk}, \mu_{pqi} + \mu_{pqj} + \mu_{pqk}] = 0, \ \ [\mu_{ijk}, \mu_{pqr}] = 0.$$

(with the obvious action of S_{n-1} which extends to an action of S_n).

From
Title: The cohomology ring of the real locus of the moduli space of stable curves of genus 0 with marked points
Authors: Pavel Etingof, Andre Henriques, Joel Kamnitzer, Eric Rains

Pasted from <http://front.math.ucdavis.edu/math/0507514>

Cyclic parathetizations:

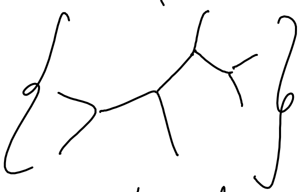
$AB = BA$
$(AB)C = A(BC)$ } only at top level

$CP_2 = \bullet$

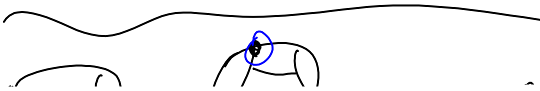
$CP_3 = \bullet$

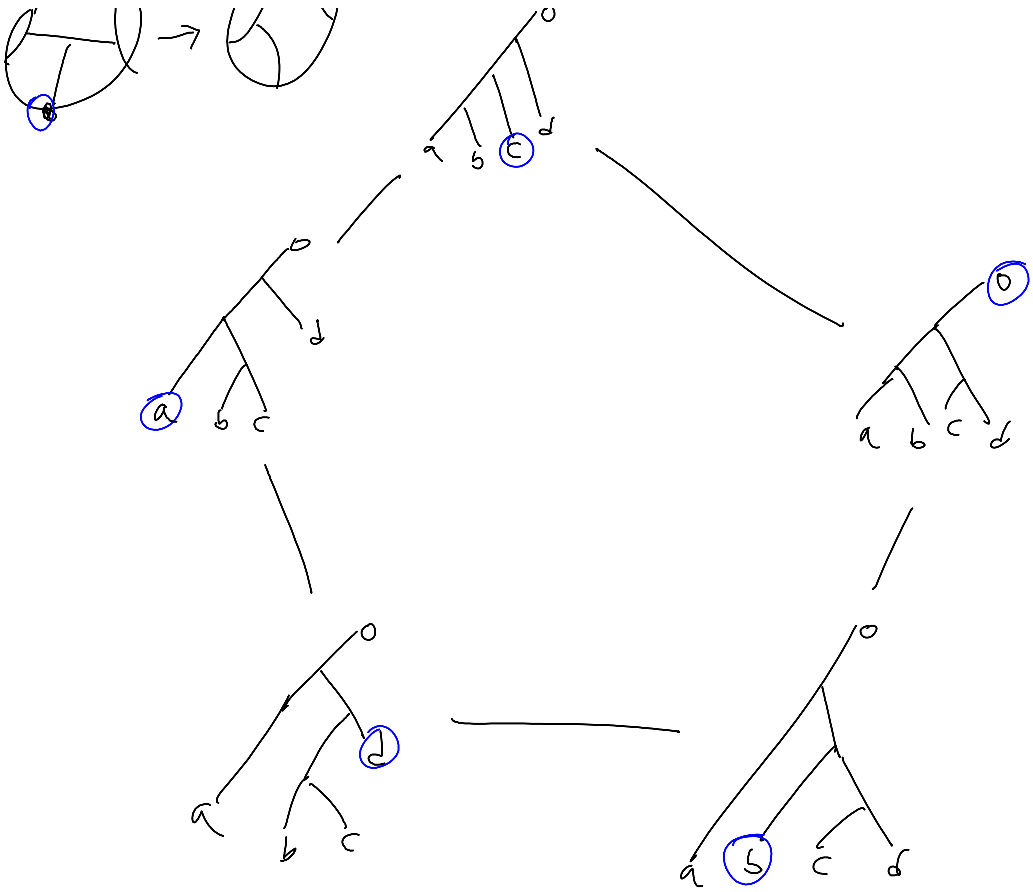
$CP_4 = \{(ab)(cd) = a(b(cd)) =$

$\|$



unrooted planar binary graphs (with cyclic numbering of the legs).





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