GPV Asides

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* The "GPV" I'm referring to is about "full" sub-diag formulas, including signs on the arrows. The relation with "classical" GPV is to be established.

Reasonable Speculations:

S1. Had there been a plain tangle formula for Z, GPV would follow.

S2. If $\alpha: \mathcal{A} \to \vec{\mathcal{A}}$ was injective, and we had a local universal invariant of V-knots, GPV would follow.

Challenges ✓1. Prove these.

2. Does GPV follow from the existence of "shielded" formulas for Z?

probably not 3. Does GPV imply that α is injective?

4. Can we follow GPV tracks to prove that α is injective?

Proof of S1: A universal subdiagram formula is an element $G \in \vec{\mathcal{D}} \otimes \mathcal{A}$ s.t.

U1. Given $K \in \mathcal{K}$, $\langle SK, G \rangle = Z(K)$ where S is the usual "subdiagram map", and $\langle\ ,\ \rangle$ is the "orthonormal" pairing $\vec{\mathcal{D}} \times \vec{\mathcal{D}} \to \mathbb{Q}$, extended by $\mathcal{A}$.

U2. G is supported on $D \otimes A$'s such that $\deg A \geq \deg D$.

It is clear that GPV follows from the existence of such G; Give ν with w.s. w, U1 implies

$$v(K) = w(Z(K)) = \langle SK, w(G) \rangle,$$

So $w(G)$ is a subdiagram formula for v, and by U2, it involves only subdiagrams of degree $\leq$ the type of v.

Now assuming a plain tangle formula for Z, set

$$G = \sum_{D \in \vec{\mathcal{D}}} D \otimes Z(\underbrace{S^{-1}(D)}_{\substack{\text{This only make sense} \\ \text{assuming a plain tangle} \\ \text{formula.}}})$$

U2 is clear. U1 follows as follows:

$$\langle SK, G \rangle = \langle SK, \sum D \otimes Z(S^{-1}(D)) \rangle$$
$$= Z(S^{-1}(S(K))) = Z(K).$$

(only the linearity of Z was used here).

<u>Proof of S2</u>: For the same reason as in the proof of S1, we can find $G^v \in \vec{\mathcal{D}} \otimes \mathcal{A}^v$ s.t.

1. For any $K \in v\mathcal{K}$,
$$\langle SK, G^v \rangle = Z^v(K)$$
2. G^v is supported on $D \otimes A$'s s.t. $\deg A \geq \deg D$.

Now if $\alpha: \mathcal{A} \to \mathcal{A}^v$ is injective, it has a one sided inverse $\beta: \mathcal{A}^v \to \mathcal{A}$ s.t.

$$\beta \circ \alpha = I_{\mathcal{A}}.$$

Set $G=\beta(G^v)$, and everything is easy.

Does GPV imply that α is injective?

$$\begin{tikzcd}
\mathcal{D} \arrow[r, shift left, "\bar\alpha"] \arrow[d, "\pi"'] & \vec{\mathcal{D}} \arrow[l, shift left, "\bar\beta"] \arrow[d, "\vec\pi"] \\
\mathcal{D}/4T=\mathcal{A} \arrow[r, "\alpha"'] & \vec{\mathcal{A}}=\vec{\mathcal{D}}/6T
\end{tikzcd}$$

Probably,

GPV constructs a $\bar\beta$ as on the left, for which

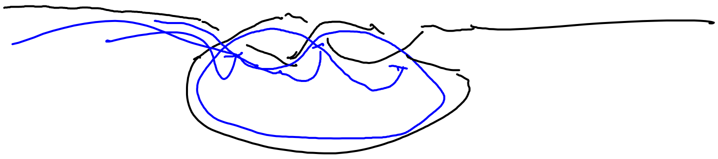
$$\pi\bar\beta\bar\alpha=\pi$$

$\Rightarrow$ No.

Does GPV follow from the existence of "shielded" formulas for Z?

Given K, is there a "canonical" choice of "escape routes" in a planar presentation of K?

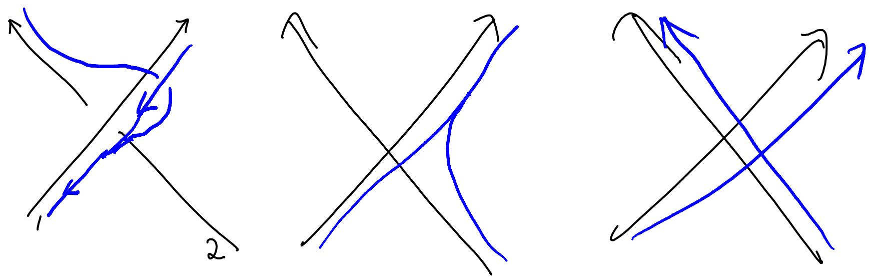
– of course – escape back along the knot itself:



<u>Problem</u> These escape routes are intersecting. To make them non-intersecting, we may need the "global" structure of the knot.

My best guess is that nonetheless, this <u>will</u> work.

Will this lead to an extension of Z to virtuals?



$$\prod_{i\in A} x_i = \prod_{i\in A}\left(1+(x_i-1)\right) = \sum_{B\subset A} \underbrace{\prod_{i\in B}(x_i-1)}_{\substack{\text{we'd like this to} \\ \text{be of degree } \geq |B|}}$$