Ghiggini on tight contact structures

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4:20 PM

M: oriented 3-manifold, ξ: tangent plane field

$\xi = \ker \alpha$, $\alpha \in \Omega^1(M)$

ξ is a contact structure if $\alpha \wedge d\alpha$ is a positive volume form on M.

$\nexists$ integral surface for ξ

<u>Problem</u> classification of contact structures up to isotopy.

<u>Thm</u> (Gray) if ξ_1 & ξ_2 are close in the C^1 sense, then they are isotopic

$\Rightarrow$ $\mathrm{Cont}(M)/\text{isotopy}$ is a discrete set.

A contact structure is "overtwisted" if

$\exists \underset{\text{disk}}{D} \hookrightarrow (M, \xi)$ s.t. $TD|_{\partial D} = \xi|_{\partial D}$

otherwise it is "tight"

<u>Thm</u> (Eliashberg) $\underset{\substack{\uparrow \\ \text{overtwisted}}}{\mathrm{Cont}_{ot}}(M)/\text{iso} \cong \dfrac{\mathrm{Dist}(M)}{\text{homotopy}}$ ← 2-plane fields on M

<u>Today</u> $-\Sigma(2,3,6n-1) = M =: Y_n =$ surgery on

<u>Thm</u> (with Van Horn-Morris)

On Y_n there are exactly $\dfrac{n(n-1)}{2}$ distinct tight contact structures.

tight contact structures. a triangular number

$i=4$

$i=0$

for Y_6

η_{ij} $\quad 0 \leq i \leq n-2 \quad -n+2+i \leq j \leq n-2-i$

$j \equiv (n-i) \ (2)$

History: 1996: Lisca & Matic distinguished the bottom row using S-W theory

~2000: Etnyre-Honda: Y_1 admits no tights.

2005: Ghiggini: top element is strongly fillable but not Stein fillable.

My usual question: Is there a combinatorial meaning to "tight contact structure"?

Another question: If I had a contact structure, what would I do with it?

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