Andersen on Groupoid Lift of Universal Invariants

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For a topological space X and a chord diagram D let

$$ch(X) := \left\langle \begin{matrix}\text{chord diagrams in } X \\ \text{i.e., } D\text{'s with homotopy} \\ \text{info}\end{matrix} \right\rangle \Big/ 4T$$

$$\mathcal{L}(X) := \langle \text{framed links in } X \rangle$$

3-manifold

$$\mathcal{L}_n(X) := \operatorname{span}\{\delta(\underbrace{\times \cdots \times}_{n})\}$$

$$gr\mathcal{L}(X) := \bigoplus_{n=0}^{\infty} \mathcal{L}_n(X)/\mathcal{L}_{n+1}(X)$$

$$\lambda : ch(X) \longrightarrow gr\mathcal{L}(X)$$ The obvious map.

<u>Def</u> $V : \mathcal{L}(X) \to ch(X)$ s.t.

$$V(\lambda(D)) = D + \text{higher order}$$

is a universal finite invariant.

Kontsevich: Such a V exists for $X = S^3$

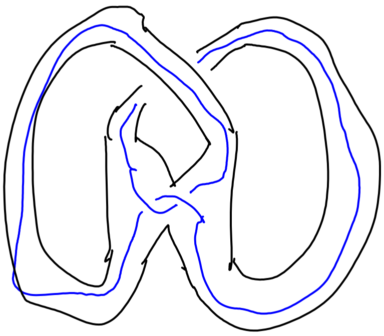
<u>AMR</u> (Andersen-Mattes-Reshetikhin)

Such a V exists for $X = \Sigma \times I$.

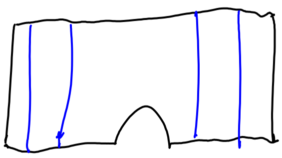
Surfaces always have just one bndry component.

<u>Pf</u>

p:



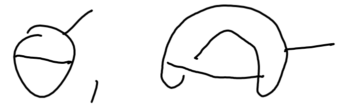
Use P.T. everywhere except add "hexagon" connectors



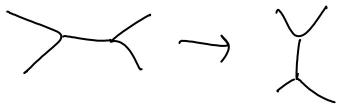
ABMP:

Let G be a unitrivalent "fat graph" graph

"Fat graph"

Let G be a unitrivalent graph 

A "fatgraph marking" of a surface Σ is an embedding of a fatgraph into Σ which is a homotopy equiv.

Make a category whose objects are fatgraphs & morphisms are 

<u>Relations</u>: 1. Involutivity,

2. Faraway commutativity.

3. Pentagon.

$$V_p : \hat{\mathcal{L}}(\Sigma) / \mathcal{L}_\infty(\Sigma) \xrightarrow{\sim} \hat{ch}(\Sigma) \Rightarrow$$ Get an action of the ptolmey groupoid on chord diagrams.

<u>Part I</u> $\mathcal{H}_\Sigma$ = free v.s. generated by homology cylinders over Σ

Has the "Goussarov–Habiro" filtration using claspers over trivalent graphs.

<u>Thm</u> If we are given a fatgraph marking G of Σ then there is a universal invariant

$$\nabla^G : \mathcal{H}_\Sigma \longrightarrow \mathcal{A}(h) \quad h = \operatorname{rank} H_1$$

(There is a nice gluing rule as well)