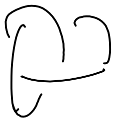
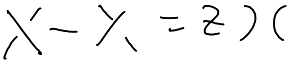
pA Talk

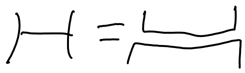
September-24-08
4:19 PM

1. The MVA:

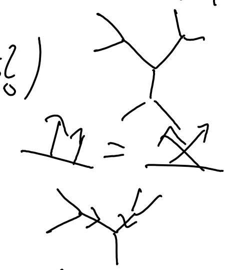
 $\longrightarrow \det(\,.\ .\ .\ .\,) =$ some poly.

2. Relations: ~~R1~~ R2, R3 4T





H. Murakami J Murakami



Naik-Stanford (only for knots?)

overXing commute

Commutators commute

3. The challenge: Verify all at max confidence & min brain utilization

$\Longrightarrow$ Need a useful & computable extention of the MVA to (virtual) tangles.

4. Virtual tangles form a circuit algebra.

5. The circuit algebra of Alexander half densities

6. The images of the generators

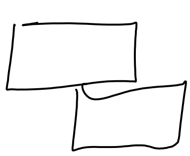
7 The program

8 sample runs

Where is it coming from?

9. Wirtinger, Fox, the determinant formula

10.

$\det\Big($  $\Big) = .\ .\ .\ .$

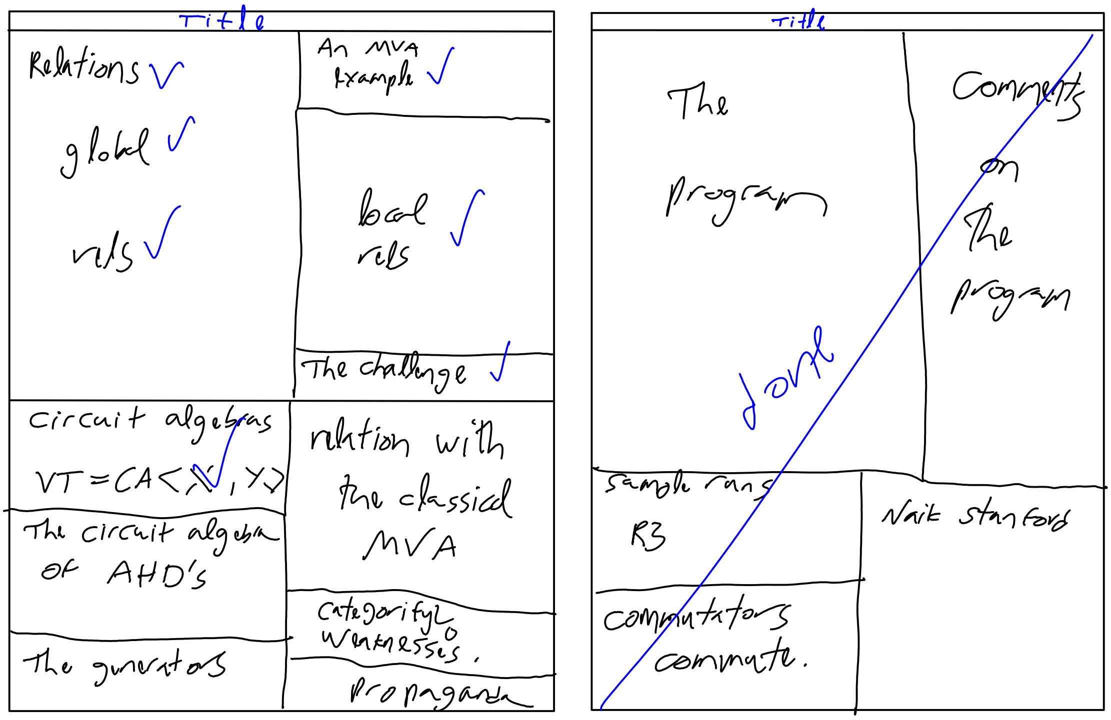
Where is it going?

11. Can you categorify this?

12 Weaknesses: Exponential, no understanding of cabling.

no obvious meaning

The handout: (use old tech - xfig etc.)



No ~WM~ !

fold at this width

```
(* Variable Equivalences *)
ReductionRules[Times[]] = {};
ReductionRules[Equal[a_, b__]] := (# -> a) & /@ {b};
ReductionRules[eqs_Times] := Join @@ (ReductionRules /@ List @@ eqs)
```

WM:

```
(* Wedge Multiply *)
WExpand[expr_] := Expand[expr /. w_W :> Signature[w]*Sort[w]];
WM[___, 0, ___] = 0;
a_ ~WM~ b_ := WExpand[Distribute[a ** b] /.  (c1_. * w1_W) ** (c2_. * w2_W) :> c1 c2 Join[w1, w2]];
WM[a_, b_, c__] := a ~WM~ WM[b, c];
```

IM:

Move this out of the html and into PA.m

```
(* Interior Multiplication *)
IM[{}, expr_] := expr;
IM[i_, w_W] := If[~~MemberQ~~[w, i], -(-1)^Position[w, i][[1, 1]]*DeleteCases[w, i], 0];
IM[{is___, i_}, w_W] := IM[{is}, IM[i, w]];
IM[is_List, expr_] := expr /. w_W :> IM[is, w]
```

FreeQ

no space

```
(* Alexander Half Densities *)
AHD[is_, -os_, eqs_, p_] := AHD[is, os, eqs, Expand[-p]];
AHD /: Reduce[AHD[is_, os_, eqs_, p_]] :=
 AHD[Sort[is], WExpand[os], eqs, WExpand[p /. ReductionRules[eqs]]];
AHD /: AHD[is1_, os1_, eqs1_, p1_] * AHD[is2_, os2_, eqs2_, p2_] := Module[{~~glued~~},
 glued = Union[Intersection[is1, List @@ os2], Intersection[is2, List @@ os1]];
 Reduce[AHD[
  Complement[Union[is1, is2], glued],
  IM[glued, os1~WM~os2],
  eqs1*eqs2 //.  eq1_Equal*eq2_Equal /;
   Intersection[List @@ eq1, List @@ eq2] =!= {} :> Union[eq1, eq2],
```

push

push

remove

spaces

change to

(is ∪ is) ∩ (os ∪ os)

```
   IM[glued, p1~WM~p2]
  ]]
 ]

   (* pA on Crossings *)
pA[Xp[i_, j_, k_, l_]] := AHD[{i, l}, W[j, k], (t[i] == t[k]) (t[j] == t[l]),
  W[l, i] + (t[i] - 1) W[l, j] - t[l] W[l, k] + W[i, j] + t[l] W[j, k] ];
pA[Xm[i_, j_, k_, l_]] := AHD[{i, j}, W[k, l], (t[i] == t[k]) (t[j] == t[l]),
  t[j] W[i, j] - t[j] W[i, l] + W[j, k] + (t[i] - 1) W[j, l] + W[k, l] ]

   (* pA on Circuit Diagrams *)
pA[cd_CircuitDiagram] := pA[cd, {}, 1];
pA[cd_CircuitDiagram, inside_, ahd_] := Module[
  {pos = First[Ordering[Length[Complement[List @@ #, inside]] & /@ cd]]},
  pA[Delete[cd, pos], Union[inside, List @@ cd[[pos]]], ahd*pA[cd[[pos]]]]
 ];
pA[CircuitDiagram[], _, ahd_] := ahd
```

Save one List, trade Intersection Join.

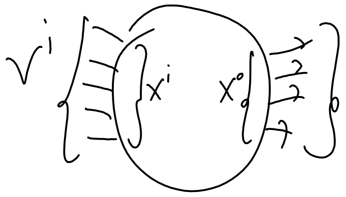
$\longrightarrow$ null

$\longrightarrow$ null

input equalities

AHD[{ } . . .]

Pasted from <http://www.math.toronto.edu/~drorbn/Talks/Sandbjerg-0810/>



V^o $\qquad V^{io}=V^i\oplus V^o$

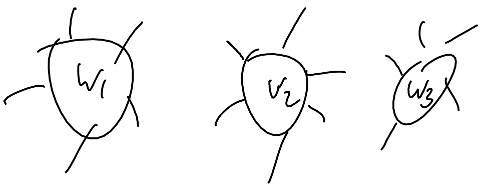
always

$A(2n)=A(X^i,X^o)=\Lambda^{1/2}(V^{io})\otimes\Lambda^{top}(V^o)$

$=A(V^i,V^o)$

$W_1\otimes W_2\in\Lambda^{1/2}(V_1^{io}\oplus V_2^{io})\otimes\Lambda^{top}(V_1^o\oplus V_2^o)$ indep of order

In general



$\mapsto \otimes W_k\in\Lambda^{1/2}(\oplus V_k^{io})\otimes\Lambda^{top}(\oplus V_k^o)$

Now given $G: V^G \xrightarrow{1-1} V^i$, $V^G\subset V^o$, s.t.

$V^{io}/ := V^{io}/_{v=Gv}$ $\qquad V^o/_G := V^o/_{V^G}$

$V^i/_G := V^i/_{GV^G}$

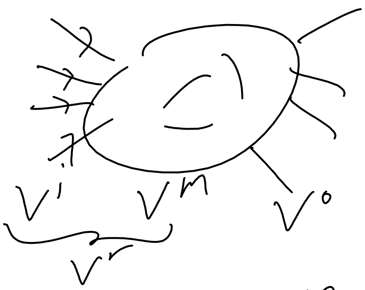
Claim we have a well defined map

claim we have a well defined map

$$A(V^i, V^o) \longrightarrow A(V^i/G, V^o/G)$$

$$0 \to V^G \xrightarrow{v \mapsto (v, -\varepsilon v)} V^{io} \longrightarrow V^{io}/G \to 0$$

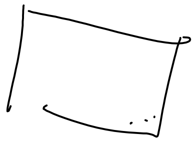
$$0 \to V^G \longrightarrow V^o \longrightarrow V^o/G \longrightarrow 0$$

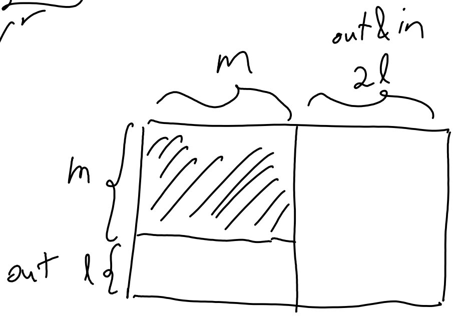


$$M: V^r \longrightarrow V^r \oplus V^o$$

$$\Lambda^{top}(V^r) \longrightarrow \Lambda^{|r|}(V^r \oplus V^o)$$

$$\subset \Lambda^*(V^r) \otimes \Lambda^*(V^o)$$





m interior arcs

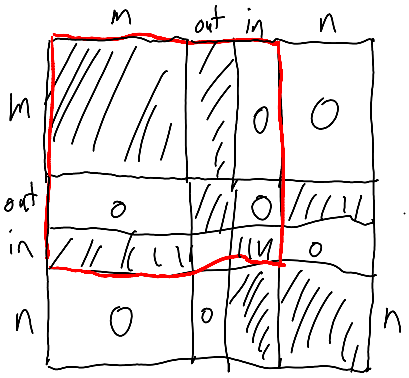
l incoming

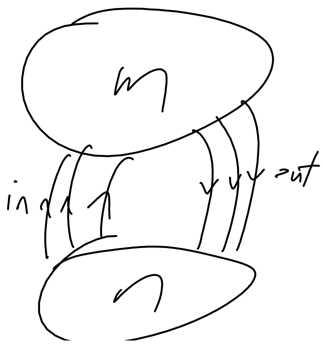
l outgoing

get a map $\Lambda^l(V^o) \to \Lambda^l(V^o \oplus V^{in})$

Applying ρA to all xings at once

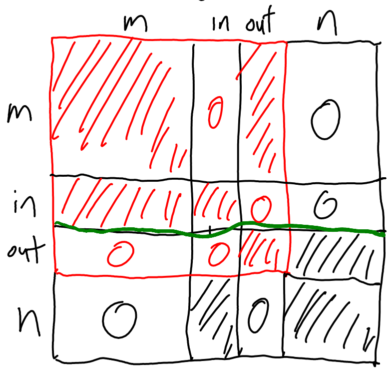
$$\Lambda^e(E) \otimes \Lambda^e\left(\begin{smallmatrix}\text{half}\\ \text{edges}\end{smallmatrix}\right) \longrightarrow \left(\Lambda^l(E) \otimes \Lambda^l(E)\right)$$





n

Alt. Arrangement



<u>Q</u> where do the minors of a matrix naturally live?

$T: V \to V$

$T: \Lambda^* V \to \Lambda^* V$