Mikhalkin: What are tropical counterparts of algebraic varieties?

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Tropical geometry: $\mathbb{T} = [-\infty, \infty)$

"$a+b$" $= \max(a,b)$ "ab" $= a+b$

(Think asymptotics: $t^a + t^b \xrightarrow[t\to\infty]{} t^{\max(a,b)}$
$t^a \cdot t^b \longrightarrow t^{a+b}$)

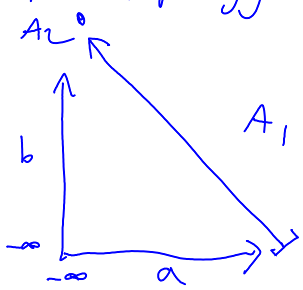
$\mathbb{TP}^n := \{(a_0 \dots a_n) : \text{not all are } -\infty\} / \text{add the same quantity to all coords}$

$\mathbb{TP}^1 = \{(0, \underset{[-\infty,\infty)}{a})\} \cup \{(-\infty, 0)\} = [-\infty, \infty]$

$=$

$\mathbb{TP}^2 = \overbrace{\{(0,a,b)\}}^{A_0} \cup \underbrace{\{(-\infty,0,b)\}}_{A_1} \cup \underbrace{\{(-\infty,-\infty,0)\}}_{A_2}$

with the topology



= a simplex.

what's a nice map $(-\infty, 0) \longrightarrow (-\infty, \infty)$?

ans $x \longmapsto \log(-x)$

$(0,1) \longrightarrow (-\infty, \infty)$

$\underset{(0,1)}{x} \mapsto \underset{(-\infty,\infty)}{\log x} \to \log(-\log x)$

map $\mathbb{TP}^n \to \Delta^n$ by

$(a_0 \dots a_n) \mapsto (a_1 - a_0, a_2 - a_1, \dots a_n - a_{n-1})$

$\mapsto$?