Q Even if finite group theory what rep-theoretic
op corresponds to the convolution of characters?

Q Is the convolution of two Ad-invariant functions
again Ad-invariant?   Yes:

$$(f * g)(z) = \int dx \; f(x) \, g(x^{-1} z)$$

$$(f * g)(h^{-1} z h) = \int dx \; f(x) \, g(x^{-1} h^{-1} z h) = \qquad \left(\begin{array}{l} s.t \\ y = h x h^{-1} \\ x = h^{-1} y h \end{array} \right)$$

$$= \int dy \; f(h^{-1} y h) \, g(h^{-1} y^{-1} z h)$$

$$= \int dy \; f(y) \, g(y^{-1} z) = f * g(z)$$