

Zlil Sela on Growth of Groups

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Thm (Mikser, Wolf, 68) Let Γ be a f.g. solvable group. Then either Γ has exponential growth or (xor) it is virtually nilpotent.

(Def G is virtually $X \iff G$ has a finite index which is X)

Thm (Tits 72, Br-Gabai, Shalom 98) If Γ is f.g. linear, then either Γ is virtually solvable or Γ contains a non-Abelian free subgroup.

Thm (Gromov, 81) a f.g. group has poly growth iff it is virtually nilpotent.

Thm (Bass) if Γ is virtually nilpotent, $\Gamma = \Gamma_0 \supset \Gamma_1 \supset \Gamma_2 \dots$ is the lower central series, then

$$d = \sum_{i=0}^{\infty} (i+1) \text{rk } \Gamma_i / \Gamma_{i+1}$$

is the degree of the polynomial growth