

"Relation 1 "

AT[{Pos[1, 2, 3, 4]}, 0, 2, {{1, 3, 2, 4}}] - AT[{Neg[1, 2, 3, 4]}, 0, 2, {{1, 3, 2, 4}}] ===
Expand[(t[1] - 1) * AT[{P[4, 1], P[3, 2]}, 0, 2, {{1, 3, 2, 4}}]]

Relation 1

True

"Relation 2"

AT[{CD[1, 3, 2, 4]}, 0, 2, {{1, 3, 2, 4}}] ===
Expand[t[1] * AT[{P[1, 4], P[2, 3]}, 0, 2, {{1, 3, 2, 4}}]]

Relation 2

True

"Relation 6 - 4 Y"

AT[{CD[2, 1, 4, 8], CD[5, 9, 1, 6], HBlob[3, 7]}, 1, 5, {{3, 7}, {8, 4}, {9, 5}, {6, 1, 2}}] -
AT[{CD[5, 1, 3, 7], CD[4, 8, 1, 9], HBlob[2, 6]}, 1, 5, {{3, 7}, {8, 4}, {9, 1, 5}, {6, 2}}] +
AT[{CD[4, 1, 2, 6], CD[3, 7, 1, 8], HBlob[5, 9]}, 1, 5, {{3, 7}, {8, 1, 4}, {9, 5}, {6, 2}}] -
AT[{CD[3, 1, 5, 9], CD[2, 6, 1, 7], HBlob[4, 8]}, 1, 5, {{3, 1, 7}, {8, 4}, {9, 5}, {6, 2}}] -
(AT[{CD[2, 1, 5, 9], CD[4, 8, 1, 6], HBlob[3, 7]}, 1, 5, {{3, 7}, {8, 4}, {9, 5}, {6, 1, 2}}] -
AT[{CD[5, 1, 4, 8], CD[3, 7, 1, 9], HBlob[2, 6]}, 1, 5, {{3, 7}, {8, 4}, {9, 1, 5}, {6, 2}}] +
AT[{CD[4, 1, 3, 7], CD[2, 6, 1, 8], HBlob[5, 9]}, 1, 5, {{3, 7}, {8, 1, 4}, {9, 5}, {6, 2}}] -
AT[{CD[3, 1, 2, 6], CD[5, 9, 1, 7], HBlob[4, 8]}, 1, 5, {{3, 1, 7}, {8, 4}, {9, 5}, {6, 2}}])

Relation 6 - 4 Y

0

"Relation 7 - Welded Knots"

AT[{Pos[7, 1, 3, 6], Pos[3, 2, 4, 5], P[1, 9], P[2, 8]},
3, 6, {{5, 2, 8}, {6, 1, 9}, {7, 3, 4}}] ===
AT[{Pos[7, 8, 1, 3], Pos[1, 9, 4, 2], P[5, 3], P[6, 2]}, 3, 6, {{5, 3, 8}, {6, 2, 9}, {7, 1, 4}}]

Relation 7 - Welded Knots

True

"Relation 9 - Virtual Tails Commute"

AT[{Arr[3, 7, 2, 1], Arr[4, 5, 1, 6]}, 1, 4, {{1, 2, 6}, {3, 7}, {4, 5}}] ===
AT[{Arr[3, 7, 1, 6], Arr[4, 5, 2, 1]}, 1, 4, {{1, 2, 6}, {3, 7}, {4, 5}}]

Relation 9 - Virtual Tails Commute

True

"Relation 13 - No internal vertices for arrow diagrams"

```

AT[{Arr[1, 9, 8, 13], Arr[2, 1, 7, 12], Arr[3, 2, 6, 11], Arr[4, 3, 5, 10]},
  3, 8, {{1, 2, 3, 4, 9}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
AT[{Arr[1, 9, 7, 12], Arr[2, 1, 8, 13], Arr[3, 2, 6, 11], Arr[4, 3, 5, 10]},
  3, 8, {{1, 2, 3, 4, 9}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
AT[{Arr[1, 9, 8, 13], Arr[2, 1, 7, 12], Arr[3, 2, 5, 10], Arr[4, 3, 6, 11]},
  3, 8, {{1, 2, 3, 4, 9}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] +
AT[{Arr[1, 9, 7, 12], Arr[2, 1, 8, 13], Arr[3, 2, 5, 10], Arr[4, 3, 6, 11]},
  3, 8, {{1, 2, 3, 4, 9}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
(AT[{Arr[1, 9, 6, 11], Arr[2, 1, 5, 10], Arr[3, 2, 8, 13], Arr[4, 3, 7, 12]},
  3, 8, {{1, 2, 3, 4, 9}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
AT[{Arr[1, 9, 5, 10], Arr[2, 1, 6, 11], Arr[3, 2, 8, 13], Arr[4, 3, 7, 12]},
  3, 8, {{1, 2, 3, 4, 9}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
AT[{Arr[1, 9, 6, 11], Arr[2, 1, 5, 10], Arr[3, 2, 7, 12], Arr[4, 3, 8, 13]},
  3, 8, {{1, 2, 3, 4, 9}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] +
AT[{Arr[1, 9, 5, 10], Arr[2, 1, 6, 11], Arr[3, 2, 7, 12], Arr[4, 3, 8, 13]},
  3, 8, {{1, 2, 3, 4, 9}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}])

```

Relation 13 - No internal vertices for arrow diagrams

0

"Alternate Version for 13 - Not 0!"

```

AT[{Arr[1, 9, 8, 13], Arr[2, 14, 7, 12], Arr[3, 15, 6, 11], Arr[4, 16, 5, 10]},
  0, 8, {{1, 9}, {2, 14}, {3, 15}, {4, 16}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
AT[{Arr[1, 9, 7, 12], Arr[2, 14, 8, 13], Arr[3, 15, 6, 11], Arr[4, 16, 5, 10]},
  0, 8, {{1, 9}, {2, 14}, {3, 15}, {4, 16}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
AT[{Arr[1, 9, 8, 13], Arr[2, 14, 7, 12], Arr[3, 15, 5, 10], Arr[4, 16, 6, 11]},
  0, 8, {{1, 9}, {2, 14}, {3, 15}, {4, 16}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] +
AT[{Arr[1, 9, 7, 12], Arr[2, 14, 8, 13], Arr[3, 15, 5, 10], Arr[4, 16, 6, 11]},
  0, 8, {{1, 9}, {2, 14}, {3, 15}, {4, 16}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
(AT[{Arr[1, 9, 6, 11], Arr[2, 14, 5, 10], Arr[3, 15, 8, 13], Arr[4, 16, 7, 12]},
  0, 8, {{1, 9}, {2, 14}, {3, 15}, {4, 16}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
AT[{Arr[1, 9, 5, 10], Arr[2, 14, 6, 11], Arr[3, 15, 8, 13], Arr[4, 16, 7, 12]},
  0, 8, {{1, 9}, {2, 14}, {3, 15}, {4, 16}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] -
AT[{Arr[1, 9, 6, 11], Arr[2, 14, 5, 10], Arr[3, 15, 7, 12], Arr[4, 16, 8, 13]},
  0, 8, {{1, 9}, {2, 14}, {3, 15}, {4, 16}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}] +
AT[{Arr[1, 9, 5, 10], Arr[2, 14, 6, 11], Arr[3, 15, 7, 12], Arr[4, 16, 8, 13]}, 0,
  8, {{1, 9}, {2, 14}, {3, 15}, {4, 16}, {8, 13}, {7, 12}, {6, 11}, {5, 10}}]) == 0

```

Alternate Version for 13 - Not 0!

False

"The double delta move - Stanford-Naik - Relation 15 - can't compute run out of memory"

```
t[1] = x; t[2] = x; t[3] = x; t[4] = x; t[5] = x; t[6] = x;
```

```
AT[{Pos[27, 7, 6, 1], Pos[28, 29, 5, 7], Pos[5, 30, 4, 8], Pos[6, 8, 3, 2],
  Pos[11, 4, 19, 15], Pos[10, 3, 11, 14], Pos[18, 15, 20, 21], Pos[17, 14, 15, 18],
  Neg[26, 1, 9, 12], Neg[9, 2, 10, 13], Neg[16, 13, 17, 23], Neg[25, 12, 16, 24]},
  18, 24, {{19, 11, 10, 9, 26}, {20, 18, 17, 16, 25}, {21, 15, 4, 5, 28},
  {22, 14, 3, 6, 27}, {23, 13, 2, 8, 30}, {24, 12, 1, 7, 29}}] ==
AT[{Pos[26, 27, 1, 4], Pos[1, 28, 2, 5], Pos[8, 5, 9, 12], Pos[25, 4, 8, 11],
  Neg[2, 29, 3, 6], Neg[3, 30, 19, 7], Neg[10, 7, 20, 14], Neg[9, 6, 10, 13],
  Pos[12, 13, 16, 15], Pos[16, 14, 21, 17], Pos[11, 15, 18, 24], Pos[18, 17, 22, 23]},
  18, 24, {{19, 26, 1, 2, 3}, {25, 8, 9, 10, 20}, {27, 4, 11, 18, 22},
  {28, 5, 12, 16, 21}, {29, 6, 13, 15, 24}, {30, 7, 14, 17, 23}}]
```

"What about Welded knots?"

```
AT[{Neg[9, 10, 1, 6], Neg[1, 11, 2, 5], Neg[2, 12, 3, 8], Neg[3, 13, 4, 7]},
  3, 8, {{9, 1, 2, 3, 4}, {10, 6}, {5, 11}, {12, 8}, {7, 13}}] ==
AT[{Neg[9, 12, 1, 8], Neg[1, 13, 2, 7], Neg[2, 10, 3, 6], Neg[3, 11, 4, 5]},
  3, 8, {{9, 1, 2, 3, 4}, {10, 6}, {5, 11}, {12, 8}, {7, 13}}]
```

What about Welded knots?

False