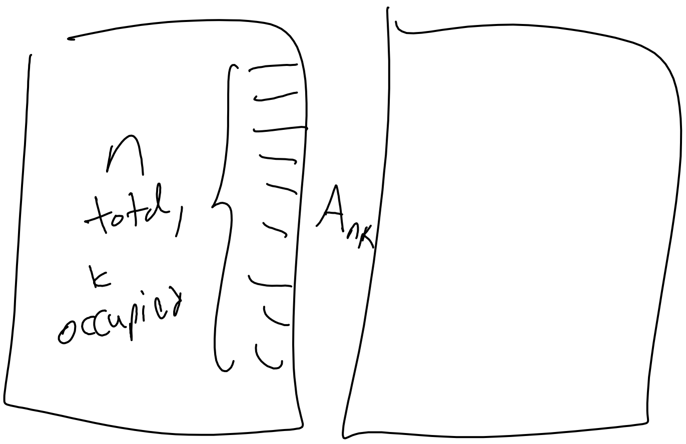
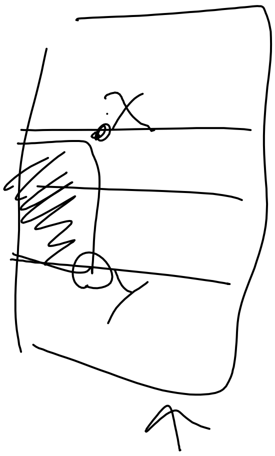
Funny algebra in HFK

June-03-08
3:00 PM





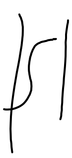
type A module

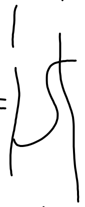
gen. over $\mathbb{F}_2$ by matchings M

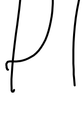
d counts internal rects
partial rects in alg. structure

type D module
$A \otimes_I M$

d counts partial rectangles

$p \cdot$  $= q$

$dx =$  y

$d(p \otimes x) = p \otimes$  $y = q \otimes y$

$$d(am) = a\,dm + (da)m \qquad d^2 m = 0$$

$$d(x\otimes y) = dx\otimes y + x\otimes dy$$

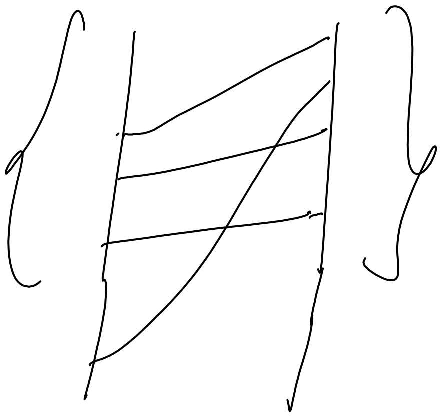
$$d(xa\otimes y) \overset{?}{=} d(x\otimes ay)$$

$$\cancel{dx\cdot a}\otimes y + \cancel{x\cdot da}\otimes y + x\cdot a\otimes dy \overset{?}{=} \cancel{dx\otimes a}y + \cancel{x\otimes da}\cdot y + x\otimes a\cdot dy$$

"$M\otimes_A N = \dfrac{MN}{A}$"

$A = \mathbb{F}_2\langle 1, \epsilon, d\epsilon\rangle$

$\epsilon^2 = 0$, $\epsilon d\epsilon = (d\epsilon)^2 = 0$

$A =$ 

deg = # of xings

obvious composition, except if two strands cross twice, $= 0$.

$d = \sum$ all ways of smoothing a xing

Claim If (M,d) is a left A module w/ $d^2=0$ & (N,d) is a right R-module s.t. $d^2=0$ on $N\otimes A$, then

(given an R-algebra A.)

$(N\otimes_R A)\otimes_A M$ has $d^2=0$

$\cong$

$N\otimes_R M$

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setup: A an DGA over R

N is a left R module with

"D type" $d: N \longrightarrow A \otimes_R N$ s.t.

$$N \xrightarrow{d} A \otimes N \xrightarrow{d\otimes 1 + 1 \otimes d} A \otimes N \text{ is } 0.$$

"A type" M is a right A-DGA-module, then

$$M \otimes_A (A \otimes N) = M \otimes_R N \text{ is a}$$

chain complex.

Q: Do matrix factorizations fit in this setup?

N: $N_0 \xrightarrow{\delta} N_1 \xrightarrow{\delta} N_0$ $\qquad \delta^2 = W$

A: e_0, e_1 : idempotents, $e_1 d = d e_0$, $e_0 d = d e_1$

$\langle e_0, e_1, \delta_e \rangle$: $\delta_e^2 = -W I$ (maybe the e_i's are unnecessary)

$d: n \longmapsto \delta_e \otimes n + 1 \otimes \delta n$

$d^2: n \longmapsto \delta_e \otimes n + 1 \otimes \delta n \mapsto 0$ ✓

Clearly M is an A-module!

P, Q $\quad [P, Q] = \lambda^2$ $\qquad P^n Q^m \lambda^d$

$PQP^{-1}Q^{-1}$