Alexander - the exact formula

June-24-08
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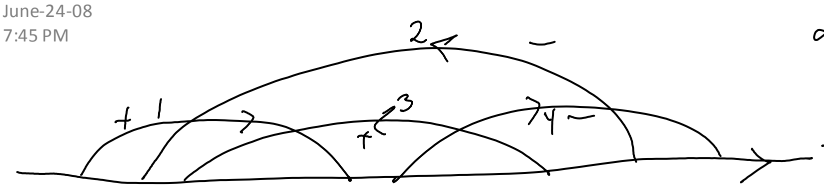


a_i: arrow #i

d_i: the direction of a_i

s_i: the sign of a_i

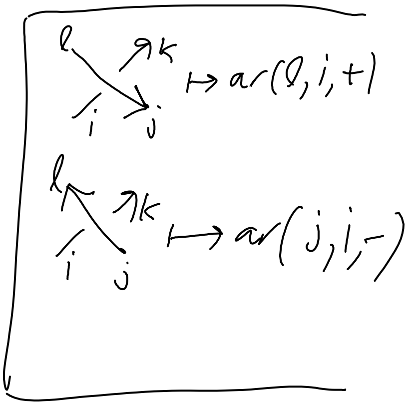
both in $\{\pm\}$

Given a Gauss diagram D, let $A=A(D)$ be the matrix with

$$A_{ij} = \begin{cases} d_i(e^{s_i x}-1) & \text{if } i\neq j \text{ and } a_j \text{ ends within the span of } a_i \\ 0 & \text{otherwise} \end{cases}$$

So for the example above,

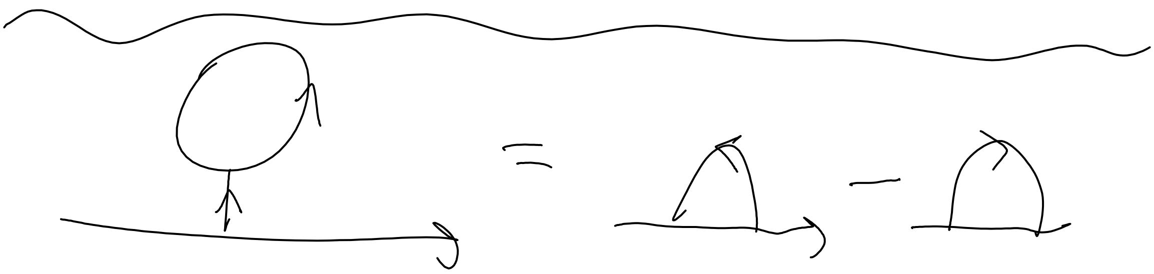
$$A = \begin{pmatrix} 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

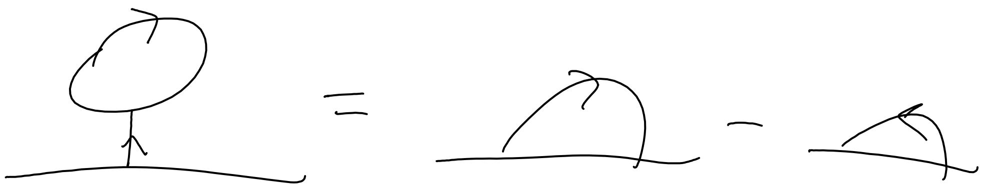


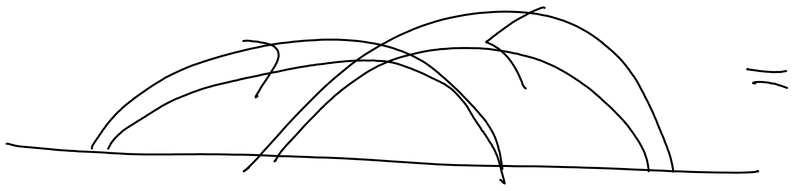
Then $\log_v Z(D) = tr\left[(I-(e^x-1)A)^{-1} - I\right]$

where x^n is interpreted as the n-wheel.

$$\left(\left(\frac{1}{I-xA}\right)_{11} = \frac{\det(I-xA)_{rest}}{\det(I-xA)} \ \cdots \right)$$







$$\Downarrow \log z$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

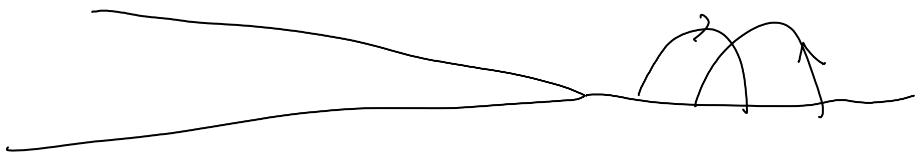
?

$$x\sum_{k=1}^{\infty}\left(e^{x}-1\right)^{k} = x\,\frac{e^{x}-1}{1-(e^{x}-1)} = x\,\frac{e^{x}-1}{2-e^{x}}$$

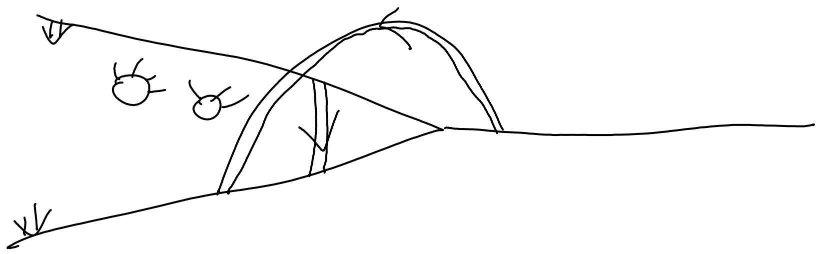
or $x\,\dfrac{1-e^{x}}{e^{x}}$

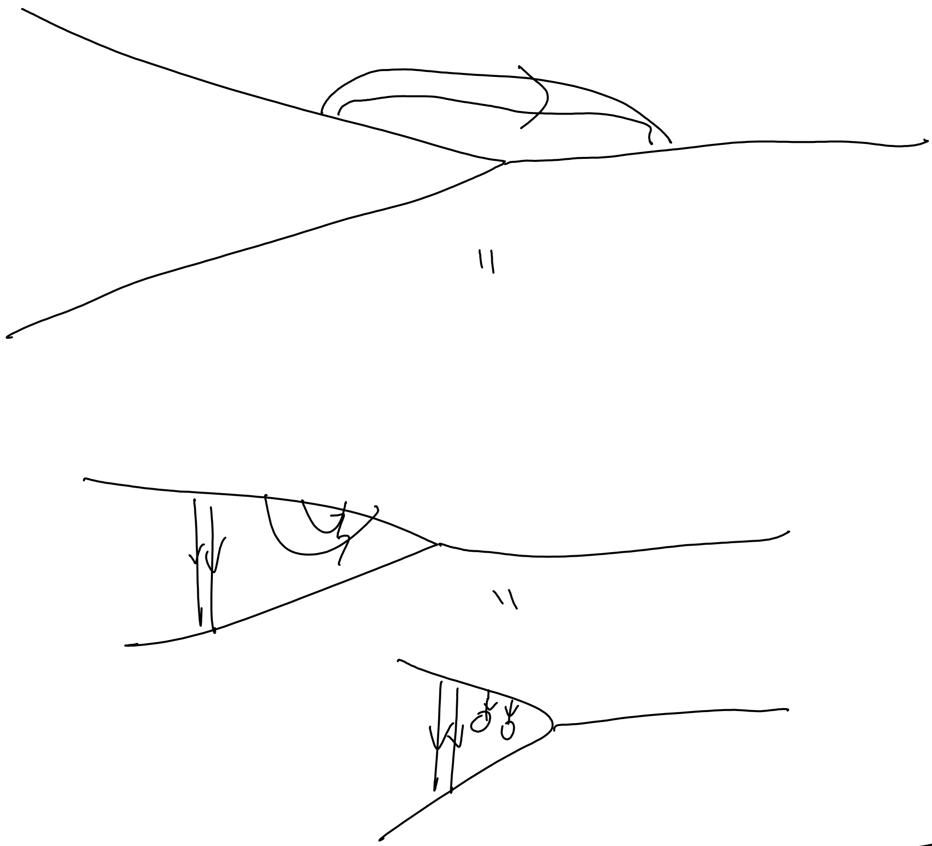
$\shortparallel$

$x\left(e^{-x}-1\right)$



$\downarrow$





$$\sum \frac{e^{nx}-1}{nx}(xy)^n = \sum \frac{e^{nx}-1}{n} y^n x^{n-1}$$

$=$