

Pensieve Header: Part of the Fiedler project.

```
<< KnotTheory`
```

Loading KnotTheory` version of January 18, 2008, 18:17:28.7446.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
Perm /: Perm[p1___] ** Perm[p2___] := Perm[p2][[p1]];
Perm[BR[n_, {}]] := Perm @@ Range[n];
Perm[BR[n_, {xs___, x_}]] := Module[{a = Abs[x]},
  Perm[BR[n, {xs}]] /. {a -> a + 1, a + 1 -> a}
];
TwistedDouble[BR[w_, br_List]] := BR[2 w, Flatten[{
  br /. {j_Integer => {2 j + {0, -1, 1, 0}}},
  Table[-Sign[Total[Sign@br]], {2 * Abs[Total[Sign@br]]}], {1}
]]];
Phi3Table[BR[n_, xings_List], k_] := Module[
  {l = Length[xings], j, lft, rgt, jj, perm, tt},
  Table[
    j = Abs[xings[[i]]];
    lft = Join[
      -Range[n - k + 1, n],
      Take[xings, i - 1],
      Range[n, 2 + j, -1]
    ];
    rgt = Join[
      Range[j - 1, 1, -1],
      Drop[xings, i] /. {jj_Integer => jj + Sign[jj]},
      -Range[2, n - k]
    ];
    perm = Perm[BR[n + 1, Join[lft, {j, j + 1, j}, rgt]]];
    tt = Perm[BR[n + 1, rgt]][[If[xings[[i]] > 0, j + 2, j + 1]]];
    While[tt > n - k, tt = perm[[tt]];
    Expand[t[tt] * (
      BR[n + 1, Join[lft, {TP[j]}, rgt]]
    )],
    {i, l}
  ]
];
b = Phi3Table[TwistedDouble[BR[Knot[4, 1]]], 5][[10]] / t[1]
```

KnotTheory::credits: The minimum braids representing the knots wit ... by Thomas Gittings. See arXiv:math.GT/0401051.

```
BR[7,
  {-2, -3, -4, -5, -6, -2, -3, -1, -2, 4, 3, 5, 4, -2, 6, 5, TP[3], 2, 1, -2, -3, 5, 4, 6, 5, 2}]
{n, 1} = List @@ b; len = Length[1];
```

```

Module[
  {t},
  Clear[kt];
  kt[s = 1, i = 0] = t = 0;
  While[{s, i} ≠ {1, len},
    If[(++i) > len, kt[s, i] = t; i = 0];
    l[[i]] /. {
      j_Integer /; s == Abs[j] => (kt[++s, i] = ++t),
      j_Integer /; s == (1 + Abs[j]) => (kt[--s, i] = ++t),
      TP[j_] /; s == Abs[j] => (kt[s += 2, i] = ++t),
      TP[j_] /; s == Abs[j] + 1 => (kt[s, i] = ++t),
      TP[j_] /; s == Abs[j] + 2 => (kt[s -= 2, i] = ++t),
      _ => (kt[s, i] = t)
    }
  ];
  tpi = Position[l, _TP][[1, 1]];
  tpj = Abs[l[[tpi, 1]]]; tps = Sign[l[[tpi, 1]]];
  kt[j_] := kt[j, tpi - 1];
  tp = If[s > 0,
    TP[kt[tpj + 2], kt[tpj + 1], kt[tpj]],
    TP[kt[tpj + 2], kt[tpj], kt[tpj + 1]]
  ];
  gds = Delete[Table[
    j = Abs[l[[i]]];
    If[l[[i]] < 0,
      X[kt[j + 1, i - 1], kt[j, i - 1]],
      X[kt[j, i - 1], kt[j + 1, i - 1]]
    ],
    {i, len}
  ], tpi] * tp;
  gds = (# /. Thread[Sort[Cases[#, _Integer, Infinity]] -> {1, 2, 3, 4, 5}]) & /@ gds

  {TP[5, 3, 2] X[1, 4], TP[5, 3, 2] X[1, 4], TP[5, 3, 2] X[1, 4],
  TP[5, 3, 2] X[1, 4], TP[5, 3, 1] X[2, 4], TP[5, 4, 3] X[1, 2],
  TP[5, 4, 3] X[2, 1], TP[5, 4, 3] X[2, 1], TP[5, 4, 3] X[2, 1],
  TP[5, 4, 3] X[2, 1], TP[5, 4, 3] X[1, 2], TP[5, 4, 2] X[1, 3], TP[5, 4, 2] X[1, 3],
  TP[5, 4, 3] X[1, 2], TP[5, 3, 2] X[1, 4], TP[5, 3, 2] X[1, 4], TP[4, 3, 2] X[1, 5],
  TP[4, 3, 2] X[1, 5], TP[5, 4, 3] X[2, 1], TP[5, 3, 2] X[4, 1], TP[5, 4, 2] X[3, 1],
  TP[5, 4, 3] X[2, 1], TP[5, 4, 2] X[3, 1], TP[5, 4, 3] X[1, 2], TP[5, 3, 2] X[1, 4]}

  ? MemberQ

```

MemberQ[list, form] returns True if an element of list matches form, and False otherwise.

MemberQ[list, form, levelspec] tests all parts of list specified by levelspec. >>

```

CyclicQ[a_, b_, c_, d_] := If[MemberQ[
  {{1, 2, 3, 4}, {2, 3, 4, 1}, {3, 4, 1, 2}, {4, 1, 2, 3}},
  Ordering[{a, b, c, d}]
], 1, 0];
gds /. TP[t1_, t2_, t3_] X[t4_, t5_] => {
  {t1, t2, t3, t4, t5},
  eta = If[Signature[{t1, t2, t3}] == 1, 0, 1],
  alpha1 = CyclicQ[t1, t3, t5, t4],
  alpha2 = CyclicQ[t1, t4, t5, t3],
  alpha3 = If[eta == 0, CyclicQ[t1, t5, t3, t4], CyclicQ[t1, t4, t3, t5]],
  alpha1 + alpha2 + alpha3
}
{{{5, 3, 2, 1, 4}, 1, 0, 0, 1, 1}, {{5, 3, 2, 1, 4}, 1, 0, 0, 1, 1},
 {{5, 3, 2, 1, 4}, 1, 0, 0, 1, 1}, {{5, 3, 2, 1, 4}, 1, 0, 0, 1, 1},
 {{5, 3, 1, 2, 4}, 1, 0, 0, 0, 0}, {{5, 4, 3, 1, 2}, 1, 0, 1, 0, 1}, {{5, 4, 3, 2, 1}, 1, 0, 0, 0, 0},
 {{5, 4, 3, 2, 1}, 1, 0, 0, 0, 0}, {{5, 4, 3, 2, 1}, 1, 0, 0, 0, 0}, {{5, 4, 3, 2, 1}, 1, 0, 0, 0, 0},
 {{5, 4, 3, 1, 2}, 1, 0, 1, 0, 1}, {{5, 4, 2, 1, 3}, 1, 0, 0, 1, 1}, {{5, 4, 2, 1, 3}, 1, 0, 0, 1, 1},
 {{5, 4, 3, 1, 2}, 1, 0, 1, 0, 1}, {{5, 3, 2, 1, 4}, 1, 0, 0, 1, 1}, {{5, 3, 2, 1, 4}, 1, 0, 0, 1, 1},
 {{4, 3, 2, 1, 5}, 1, 0, 0, 0, 0}, {{4, 3, 2, 1, 5}, 1, 0, 0, 0, 0}, {{5, 4, 3, 2, 1}, 1, 0, 0, 0, 0},
 {{5, 3, 2, 4, 1}, 1, 0, 0, 0, 0}, {{5, 4, 2, 3, 1}, 1, 0, 0, 0, 0}, {{5, 4, 3, 2, 1}, 1, 0, 0, 0, 0},
 {{5, 4, 2, 3, 1}, 1, 0, 0, 0, 0}, {{5, 4, 3, 1, 2}, 1, 0, 1, 0, 1}, {{5, 3, 2, 1, 4}, 1, 0, 0, 1, 1}}

```

```
Table[kt[s, i], {s, n}, {i, 0, Length[l]}] // MatrixForm
```

```

(
0 0 0 0 0 0 0 0 10 10 10 10 10 10 10 10 10 10 53 53 53 53 53 53 53 53
43 23 23 23 23 23 9 9 1 33 33 33 33 33 19 19 19 19 52 11 21 21 21 21 21 21 43
22 44 8 8 8 8 24 32 32 2 2 18 18 18 34 34 34 51 20 20 12 42 42 42 42 22
7 7 45 31 31 31 31 25 25 25 17 3 3 40 40 40 40 41 41 41 41 13 13 7 7 7 7
30 30 30 46 16 16 16 16 16 26 26 26 39 4 4 50 35 35 35 35 6 14 14 30 30
15 15 15 15 47 38 38 38 38 38 38 27 27 27 49 5 5 5 5 5 5 36 36 29 15 15
37 37 37 37 37 48 48 48 48 48 48 48 48 28 28 28 28 28 28 28 28 28 28 37 37 37
)

```

b

```

BR[9, {-3, -4, -5, -6, -7, -8, 2, 1, 3, 2, 2, 1, 3, 2, 4, 3, 5, 4, -6,
-7, -5, -6, 4, 3, 5, 4, 2, 1, 8, 7, 6, 5, TP[3], 2, 1, 3, -7, -8, -6, -7, -5,
-6, -4, -5, -5, -6, -4, -5, -7, -8, -6, -7, -7, -8, -6, -7, 2, 2, -2}]

```

```
BraidPlot[CollapseBraid @ BR[Knot["K11n34"]]]
```

