```
<< KnotTheory`
```

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Loading KnotTheory` version of January 18, 2008, 18:17:28.7446.
Read more at http://katlas.org/wiki/KnotTheory.
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```
wAlex[gc_GC] := {
   Aij[Ar[ti_, hi_, si_], Ar[tj_, hj_, sj_]] := If[
     ti < hj < hi || hi < hj < ti,
     (X^(si) - 1) * Sign[hi - ti],
     0
    ];
   MatrixForm[A = Outer[Aij, List @@ gc, List @@ gc]],
   Tr[Inverse[
        IdentityMatrix[Length[A]] - A
      ]] - Length[A] // Together // ExpandNumerator,
   Det[
      IdentityMatrix[Length[A]] - A
     ] // Together // ExpandNumerator
  };
wAlex[K_] := Join[{
   pd = PD[K],
   gc = GC @@ pd /. X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]],
       Ar[l, i, +1], Ar[j, i, -1]
      ]
  },
  wAlex[gc]
 ]
```

```
wAlex[BR[2, {1, 1, 1}]]
```

$$\left\{\text{PD}[X[3, 1, 4, 6], X[1, 5, 2, 4], X[5, 3, 6, 2]],\ \text{GC}[\text{Ar}[6, 3, 1], \text{Ar}[4, 1, 1], \text{Ar}[2, 5, 1]],\ \begin{pmatrix} 0 & 0 & 1-X \\ 1-X & 0 & 0 \\ -1+X & 0 & 0 \end{pmatrix},\ \frac{-2+4X-2X^2}{2-2X+X^2},\ 2-2X+X^2\right\}$$

```
wAlex[BR[3, {1, 2, 1, 2}]]
```

$$\left\{\text{PD}[X[8, 6, 1, 5], X[3, 7, 4, 6], X[4, 2, 5, 1], X[7, 3, 8, 2]],\ \text{GC}[\text{Ar}[5, 8, 1], \text{Ar}[6, 3, 1], \text{Ar}[1, 4, 1], \text{Ar}[2, 7, 1]],\ \begin{pmatrix} 0 & 0 & 0 & -1+X \\ 0 & 0 & 1-X & 0 \\ 0 & -1+X & 0 & 0 \\ 0 & -1+X & -1+X & 0 \end{pmatrix},\ \frac{-2+4X-2X^2}{2-2X+X^2},\ 2-2X+X^2\right\}$$

```
wAlex[BR[3, {1, 2, 1, 1, -1, 2}]]
```

$$\left\{\text{PD}[X[5, 3, 6, 2], X[10, 4, 11, 3], X[11, 7, 12, 6], X[7, 1, 8, 12], X[8, 1, 9, 2], X[4, 10, 5, 9]],\ \text{GC}[\text{Ar}[2, 5, 1], \text{Ar}[3, 10, 1], \text{Ar}[6, 11, 1], \text{Ar}[12, 7, 1], \text{Ar}[1, 8, -1], \text{Ar}[9, 4, 1]],\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1+X \\ -1+X & 0 & 0 & -1+X & -1+X & -1+X \\ 0 & -1+X & 0 & -1+X & -1+X & 0 \\ 0 & 1-X & 1-X & 0 & 1-X & 0 \\ -1+\frac{1}{X} & 0 & 0 & -1+\frac{1}{X} & 0 & -1+\frac{1}{X} \\ 1-X & 0 & 0 & 1-X & 1-X & 0 \end{pmatrix},\ \frac{-2+4X-2X^2}{X\left(2-2X+X^2\right)},\ 2-2X+X^2\right\}$$

```
wAlex[BR[3, {1, 2, 1, 2, -2, 2}]]
```

$$\left\{\text{PD}[\text{X}[10, 6, 11, 5], \text{X}[3, 7, 4, 6], \text{X}[4, 12, 5, 11], \text{X}[7, 1, 8, 12], \text{X}[8, 1, 9, 2], \text{X}[9, 3, 10, 2]],\ \text{GC}[\text{Ar}[5, 10, 1], \text{Ar}[6, 3, 1], \text{Ar}[11, 4, 1], \text{Ar}[12, 7, 1], \text{Ar}[1, 8, -1], \text{Ar}[2, 9, 1]],\ \begin{pmatrix} 0 & 0 & 0 & -1+X & -1+X & -1+X \\ 0 & 0 & 1-X & 0 & 0 & 0 \\ 1-X & 0 & 0 & 1-X & 1-X & 1-X \\ 1-X & 0 & 0 & 0 & 1-X & 1-X \\ 0 & -1+\frac{1}{X} & -1+\frac{1}{X} & -1+\frac{1}{X} & 0 & 0 \\ 0 & -1+X & -1+X & -1+X & -1+X & 0 \end{pmatrix},\ \frac{-7+23\,X-30\,X^2+20\,X^3-7\,X^4+X^5}{2-2\,X+X^2},\ 2-2\,X+X^2\right\}$$

```
wAlex[Knot[4, 1]]
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\left\{\text{PD}[\text{X}[4, 2, 5, 1], \text{X}[8, 6, 1, 5], \text{X}[6, 3, 7, 4], \text{X}[2, 7, 3, 8]],\ \text{GC}[\text{Ar}[1, 4, 1], \text{Ar}[5, 8, 1], \text{Ar}[3, 6, -1], \text{Ar}[7, 2, -1]],\ \begin{pmatrix} 0 & 0 & 0 & -1+X \\ 0 & 0 & -1+X & 0 \\ -1+\frac{1}{X} & 0 & 0 & 0 \\ 1-\frac{1}{X} & 0 & 1-\frac{1}{X} & 0 \end{pmatrix},\ \frac{3-7\,X+5\,X^2-X^3}{-1+2\,X},\ \frac{-1+2\,X}{X^2}\right\}$$

```
Simplify[(-1 + 2 X)/X^2 + ((-1 + 2 X)/X^2 /. X -> 1/X)]
```

$$-\frac{1-2\,X-2\,X^3+X^4}{X^2}$$

```
wAlex[Mirror[Knot[4, 1]]]
```

$$\left\{\text{PD}[\text{X}[1, 4, 2, 5], \text{X}[5, 8, 6, 1], \text{X}[3, 7, 4, 6], \text{X}[7, 3, 8, 2]],\ \text{GC}[\text{Ar}[4, 1, -1], \text{Ar}[8, 5, -1], \text{Ar}[6, 3, 1], \text{Ar}[2, 7, 1]],\ \begin{pmatrix} 0 & 0 & 1-\frac{1}{X} & 0 \\ 0 & 0 & 0 & 1-\frac{1}{X} \\ 0 & 1-X & 0 & 0 \\ 0 & -1+X & -1+X & 0 \end{pmatrix},\ \frac{5-13\,X+11\,X^2-3\,X^3}{-2+6\,X-4\,X^2+X^3},\ \frac{-2+6\,X-4\,X^2+X^3}{X}\right\}$$

```
Simplify[(-2 + 6 X - 4 X^2 + X^3)/X + ((-2 + 6 X - 4 X^2 + X^3)/X /. X -> 1/X)]
```

$$12+\frac{1}{X^2}-\frac{6}{X}-6\,X+X^2$$

wAlex[Knot[5, 1]]

$$\left\{\text{PD}[X[1, 6, 2, 7], X[3, 8, 4, 9], X[5, 10, 6, 1], X[7, 2, 8, 3], X[9, 4, 10, 5]],\ \text{GC}[\text{Ar}[6, 1, -1], \text{Ar}[8, 3, -1], \text{Ar}[10, 5, -1], \text{Ar}[2, 7, -1], \text{Ar}[4, 9, -1]],\right.$$

$$\left.\begin{pmatrix} 0 & 1-\frac{1}{X} & 1-\frac{1}{X} & 0 & 0 \\ 0 & 0 & 1-\frac{1}{X} & 1-\frac{1}{X} & 0 \\ 0 & 0 & 0 & 1-\frac{1}{X} & 1-\frac{1}{X} \\ 0 & -1+\frac{1}{X} & -1+\frac{1}{X} & 0 & 0 \\ 0 & 0 & -1+\frac{1}{X} & -1+\frac{1}{X} & 0 \end{pmatrix}, \frac{-6+12X-6X^2}{3-6X+4X^2}, \frac{3-6X+4X^2}{X^2}\right\}$$

wAlex[Knot[10, 132]]

$$\left\{\text{PD}[X[4, 2, 5, 1], X[8, 4, 9, 3], X[5, 12, 6, 13], X[15, 18, 16, 19], X[9, 16, 10, 17], X[17, 10, 18, 11], X[13, 20, 14, 1], X[19, 14, 20, 15], X[11, 6, 12, 7], X[2, 8, 3, 7]],\right.$$
$$\text{GC}[\text{Ar}[1, 4, 1], \text{Ar}[3, 8, 1], \text{Ar}[12, 5, -1], \text{Ar}[18, 15, -1], \text{Ar}[16, 9, -1], \text{Ar}[10, 17, -1], \text{Ar}[20, 13, -1], \text{Ar}[14, 19, -1], \text{Ar}[6, 11, -1], \text{Ar}[7, 2, 1]],$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1+X \\ -1+X & 0 & -1+X & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\frac{1}{X} & 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 & 1-\frac{1}{X} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\frac{1}{X} & 0 & 0 & 1-\frac{1}{X} & 0 & 1-\frac{1}{X} & 0 \\ 0 & 0 & 0 & -1+\frac{1}{X} & 0 & 0 & -1+\frac{1}{X} & 0 & -1+\frac{1}{X} & 0 \\ 0 & 0 & 0 & 1-\frac{1}{X} & 0 & 1-\frac{1}{X} & 0 & 1-\frac{1}{X} & 0 & 0 \\ 0 & 0 & 0 & -1+\frac{1}{X} & 0 & -1+\frac{1}{X} & 0 & 0 & 0 & 0 \\ 0 & -1+\frac{1}{X} & 0 & 0 & -1+\frac{1}{X} & 0 & 0 & 0 & 0 & 0 \\ 1-X & 0 & 1-X & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\frac{-7+43X-91X^2+61X^3+42X^4-78X^5+24X^6+14X^7-8X^8}{1-6X+11X^2-2X^3-13X^4+10X^5+5X^6-8X^7+3X^8},$$
$$\left.\frac{1-6X+11X^2-2X^3-13X^4+10X^5+5X^6-8X^7+3X^8}{X^6}\right\}$$

wAlex[GC[Ar[1, 3, +1], Ar[4, 2, +1]]]

$$\left\{\begin{pmatrix} 0 & -1+X \\ 1-X & 0 \end{pmatrix}, \frac{-2+4X-2X^2}{2-2X+X^2}, 2-2X+X^2\right\}$$