The Alek-Tor "divergence"

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12:21 PM

Question: what's the natural extension of div to

$$U(\mathrm{tder}_n) = \vec{A}^{wt}_n \;?$$

Alek-Tor say it's j:

$$j(gh) = j(g) + g \cdot j(h),$$

$$\frac{d}{ds} j(\exp(su))|_{s=0} = \mathrm{div}(u).$$

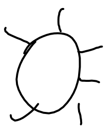
$$\begin{aligned}
\mathrm{div}[a,b] &= a\,\mathrm{div}\,b - b\,\mathrm{div}\,a \\
&= j(ab) - j(ba) \\
&= \big(j(a) + a\,j(b)\big) - \big(j(b) + b\,j(a)\big) \\
&=
\end{aligned}$$

I have to remind myself what exactly is the difference between the two exponentiations in A.

Q Is $\mathrm{tder} \oplus \mathrm{tr}$ ~~naturally~~ isomorphic to A^{wt}_{prim}?

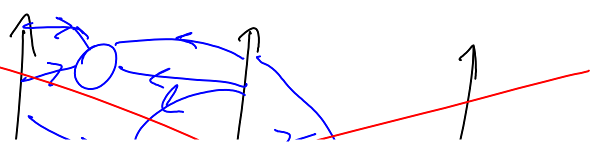
Ans Let ψ be

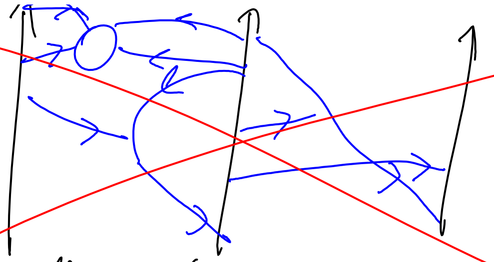
$\psi : (a_1, a_2) \longmapsto$ 

$w =$  $\longmapsto w$

YES, but a choice needs to be made, for the placement of the invariants.

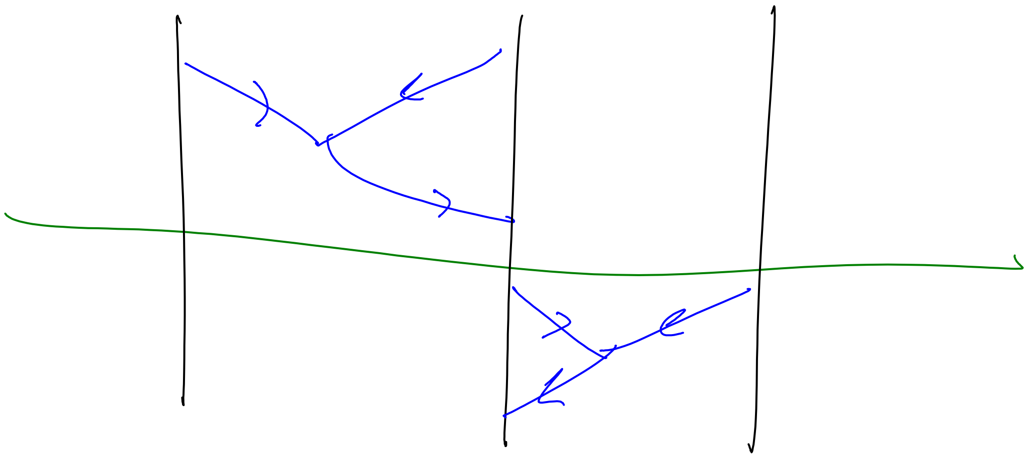
Q Is $U(\mathrm{tder} \oplus \mathrm{tr})$ isomorphic to A^{wt}?

Ans.  ψ.

Ans.  Yes

~~All roots are at the bottom.~~

No, roots end up at the bottom of each "layer":



Primitive level:

$$0 \to \{\text{wheels}\} \xrightarrow{i} \mathcal{A}^{wt} \underset{b}{\overset{t}{\leftrightarrows}} \{\text{trees}\} \to 0$$

$\mathrm{div} = i^{-1}\circ(t-b)$ Is the difference of two splittings automatically a 1-cocycle? **Yes:**

$$\begin{aligned}
\mathrm{div}[x,y] &\\
\| \quad &\\
(t-b)[x,y] &= [tx,ty]-[bx,by] \\
&= [tx,ty]-[tx,by]+[tx,by]-[bx,by] \\
&= [tx,(t-b)y]+[(t-b)x,by] \\
&= x\,\mathrm{div}\,y + y\,\mathrm{div}\,x
\end{aligned}$$

The group case:

$$\begin{tikzcd} 0 \arrow[r] & N \arrow[r, "i"] & G \arrow[r] & G/N \arrow[r] \arrow[l, bend right, "t"'] \arrow[l, bend left, "b"] & 0 \end{tikzcd}$$

Is $i^{-1}(t(x)^{-1}b(x))$ automatically a group 1-cocycle? $\underset{=}{}$ $i^{-1}(t(x^{-1})b(x)) =: j(x)$

$$t(xy)^{-1}b(xy) = t(y^{-1})t(x^{-1})b(x)b(y)$$
$$= \underbrace{t(y^{-1})b(y)}\,\underbrace{b(y^{-1})t(x^{-1})b(x)b(y)},$$
$$= j(y) + y\,j(x)$$