Special derivations within tangential derivations

April-24-08
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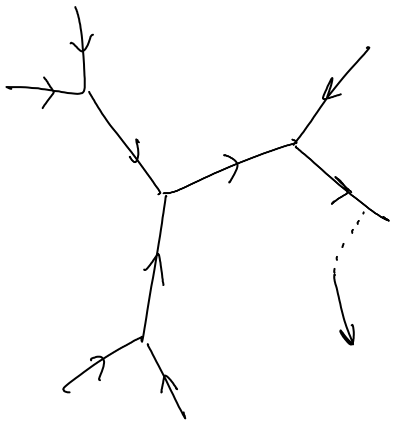
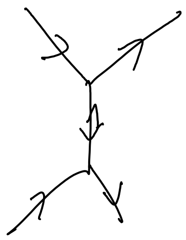
First consider the 1,1,1,...1 case.





The five-term IHX always vanishes in the co-com case.

$\alpha : \mathcal{A} \longrightarrow \vec{\mathcal{A}}^{cc}$ as usual

$\beta : \vec{\mathcal{A}}^{cc} \longrightarrow \mathcal{A}$ by

1. map  to 0
2. Forget the arrows.

well-defined!

$\alpha \circ \beta = \sum$ way of re-rooting.

$\beta \circ \alpha =$ multiplication by the leg count.

$SD := D(x_1 + \dots + x_n)$

$$\begin{tikzcd} 0 \arrow[r] & \mathcal{A}^p_n \arrow[r, "\alpha"] & \vec{\mathcal{A}}^{ccp}_n \arrow[r, "S"] \arrow[l, bend right, "\beta"'] & \mathrm{Lie}^+_n \arrow[r] \arrow[l, bend right, "\gamma"'] & 0 \end{tikzcd}$$

$w \qquad z$

! exact? !

If $\beta\alpha = I$:

$Sz = 0 \Rightarrow S(z - \alpha\beta z) = 0 \Rightarrow \exists\, z - \alpha\beta z = \alpha w$

$\Rightarrow w = \beta\alpha w = \beta z - \beta z = 0 \Rightarrow z = \alpha\beta z.$

Is $\gamma\delta + \alpha\beta = I$? [first, is there a distinguished choice for γ ?]

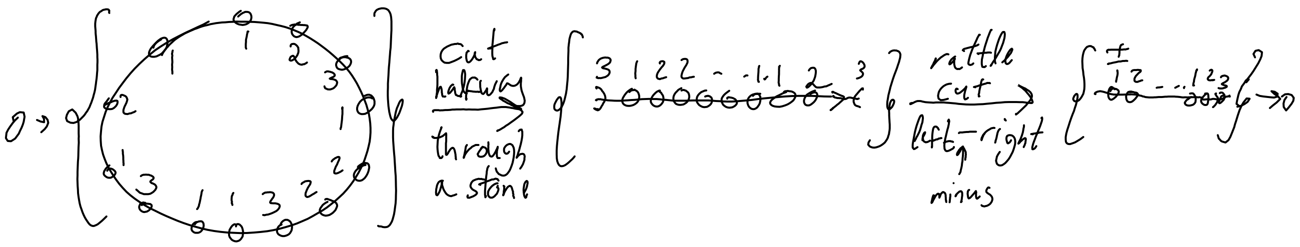
Perhaps the whole thing should be done "on the level of surfaces"?

$$0 \to \begin{Bmatrix}\text{cyclic}\\ \text{words}\end{Bmatrix} \longrightarrow \begin{Bmatrix}n\text{-tuples of}\\ \text{linear words}\end{Bmatrix} \longrightarrow \begin{Bmatrix}\text{single linear}\\ \text{word}\end{Bmatrix} \to 0$$

sum of all ways of cutting at one of the letters, tabbing(?) by that letter.

$(a_1 \dots a_n)$
$\downarrow$
$\sum [a_i, x_i]$

A different view:



Yet another view:

If M is a $\mathbb{Z}[T]/\langle T^n - 1\rangle$ module then the following sequence is exact in the middle term

$$0 \to M_T \xrightarrow{\sum_{k=0}^{n-1} T^k} M \xrightarrow{1-T} M \longrightarrow 0$$

$$M_T = M/_{m = Tm}$$

over $\mathbb{Q}$!

Proof

$$(1+x+x^2)\_\_ + (1-x)\_\_ = 1 \mod (x^n - 1)$$

$$(1+x)\tfrac{1}{2} + (1-x)\tfrac{1}{2} = 1$$

$$(1+x+x^2) - (1-x)(x+2) = 3$$

$x+2$

$$
\begin{array}{l}
\overline{x^2+x+1}\,\big|\,x-1 \\
\underline{x-1} \\
\quad 2x+1 \\
\quad \underline{2x-2} \\
\qquad 3
\end{array}
\qquad (x-1)(x+2)=x^2+x-2
$$

Do cyclic permutations act on Lie words?

$$[x,[x,y]] = \cancel{xxy} - xyx - xyx + \cancel{yxx} \longrightarrow$$

$$\xrightarrow{T} xyx - \cancel{yxx} - \cancel{yxx} + \cancel{xxy} =$$

No.

$$(1-T)[x,[x,y]] = 3yxx - 3xyx = 3[y,x]x$$

$$[x,[y,z]] \longrightarrow xyz - xzy - yzx + zyx$$

$$\xrightarrow{T} yzx - zyx - zxy + yxz$$

$$[[x,y],z] =$$