

$$Q = \{1, 2, 3\} \quad 2 \uparrow 3 = 1, \quad 3 \uparrow 2 = 3$$

$$(a \uparrow b) \uparrow c = (a \uparrow c) \uparrow (b \uparrow c)$$

if $a=1$; follows from the unit axioms.

$$b=1; \quad -||-$$

$$c=1; \quad -||-$$

So we only need to check this for $a, b, c \in \{2, 3\}$.

$$2 \ 2 \ 2 \quad \checkmark$$

no inverses!

$$2 \ 2 \ 3 \quad 1=1$$

$$2 \ 3 \ 2 \quad 1=1$$

$$2 \ 3 \ 3 \quad 1=1$$

$$3 \ 2 \ 2 \quad 3=3$$

$$3 \ 2 \ 3 \quad 3=3$$

$$3 \ 3 \ 2 \quad 3=3$$

$$3 \ 3 \ 3 \quad 3=3$$