Expansions For quandles

April-14-08
12:45 PM

An expansion: try $x \overset{Z_0}{\longmapsto} 1+\bar{x}$ for the generators,

$1 \longmapsto 1$

$$\begin{tikzcd}
(x,y) \arrow[r, "\uparrow"] \arrow[d, "Z"'] & x\uparrow y \arrow[d, "Z"] \\
(1+\bar{x}, 1+\bar{y}) \arrow[r, "\uparrow"] & (1+\bar{x})\uparrow(1+\bar{y}) \\
& (1+\bar{x})\uparrow(1+\bar{y}) \arrow[u, phantom, "//"]
\end{tikzcd}$$

Does it annihilate the quandle relation?

$$(x\uparrow y)\uparrow h \quad = \quad (x\uparrow h)\uparrow(y\uparrow h)$$

$$\downarrow$$

The only remaining term is in deg 4:

$$0 = (\bar{x}\uparrow\bar{h})\uparrow(\bar{y}\uparrow\bar{h})$$

$$[[\bar{x},\bar{h}],[\bar{y},\bar{h}]] \qquad \text{fails!}$$

<u>Questions</u>

1. Is there a different candidate?
2. Does Z_0 word "mod homotopy"?
3. Is there an alternative notion of "expansion" that does work here?
4. Is it at all true that
$$FQ_n^{proj} = FL_n \quad ?$$

For the last question, let $\mu: FQ_n \to FG_n$ be the obvious map given by $a\uparrow b \to b^{-1}ab$ and set

$$Z^{\varphi} = \log Z^{\iota_* \varphi} \circ \mu .$$

$$x \wedge y \longmapsto \log e^{-y} e^{x} e^{y} = x + [x, y] + \cdots$$

$$1 \wedge x = 1 \qquad 1 \wedge \bar{x} = 1 \wedge x - 1 \wedge 1 = 0$$