

$$\begin{aligned}
x + y - \log e^y e^x &= (1 - e^{-\text{ad } x})F + (e^{\text{ad } y} - 1)G \\
\text{tr}(\text{ad } x)\partial_x F + \text{tr}(\text{ad } y)\partial_y G &= \frac{1}{2} \text{tr} \left(\frac{\text{ad } x}{e^{\text{ad } x} - 1} + \frac{\text{ad } y}{e^{\text{ad } y} - 1} - \frac{\text{ad } z}{e^{\text{ad } z} - 1} - 1 \right) \\
&\text{with } z = \log e^x e^y
\end{aligned}$$