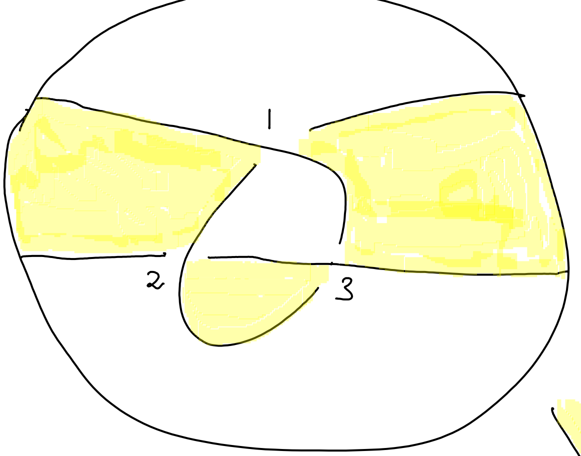
Jones of Alternating Tangles

Feb 23, 2008

$d = -A^2 - A^{-2}$

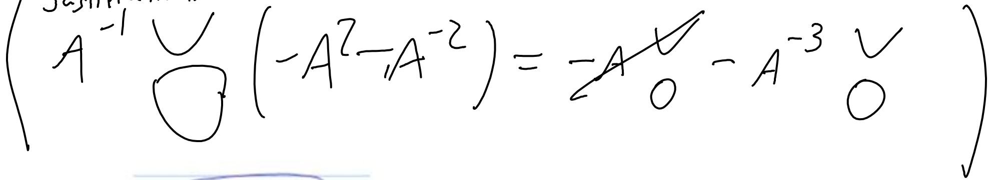


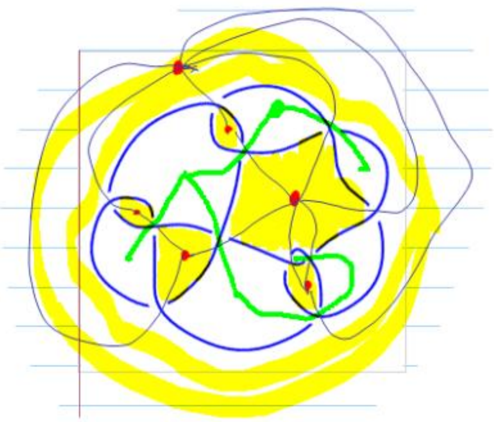
1. Consider the crossings in order, smoothing or not smoothing them one at a time.

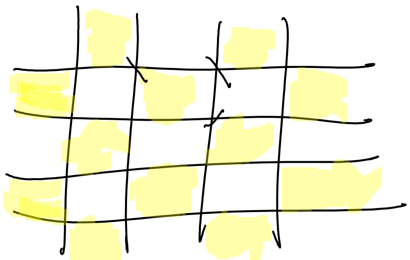
2. If smoothing either way would create a disconnected closed component, go to 3. Otherwise,

3. If one of the smoothings does create a disconnected closed component, use only the one that doesn't, with altered coefficients:

Justification:







Note: The number of white moves and the number of yellow moves are forced in advance, to be, say, w & y

FrW $\longmapsto$	A^{-1}	p
FoW $\longmapsto$	$-A^3$	$w-p$
FrY $\longmapsto$	A	q
FoY $\longmapsto$	$-A^{-3}$	$y-q$

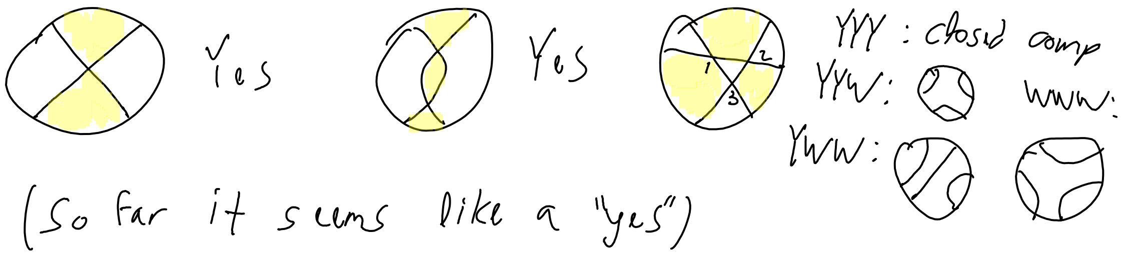
$$\longrightarrow A^{\wedge}(-p+3w-3p+q-3y+3q) = 3w-3y+4(q-p)$$

$$\cdot(-)^{\wedge}(w-p+y-q) = w+y-(p+q) \underset{\text{mod } 2}{=} w+y+(q-p)$$

So four times the exponent of A is determined by the same parameter that determines the sign, namely $q-p$. □

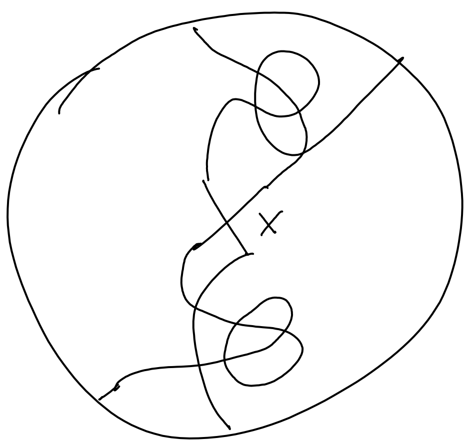
The Case of Tangles

Q Is it true that w & y are determined by the tangle and the output smoothing? (assuming no closed components)



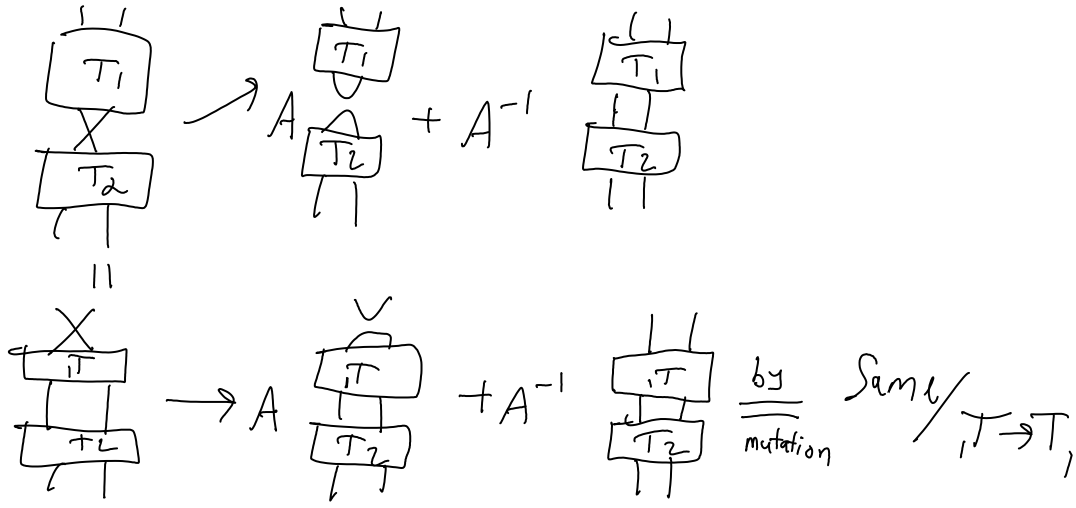
(So far it seems like a "yes")

If so, it seems that the coefficient of each specific smoothing will be alternating, and likely there will be a way to recover the lead sign from the smoothing.
Are these properties hereditary? Probably not.



what extra mileage can we get by smoothing the largely-disconnecting crossing X?

"semi-nugatory"?



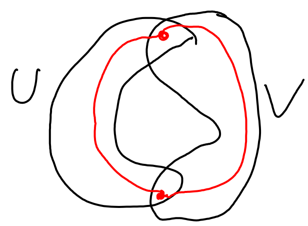
.... useless.

Q: Is there a Van-Kampen theorem when $U \cap V$ is not connected?

Ans π_1 is really a category, whose objects are points and whose morphisms are homotopy classes of paths with fixed endpoints.

$$\pi_1(U \cup V) = \pi_1(U) *_{\pi_1(U\cap V)} \pi_1(V)$$

makes perfect sense....



Question Is there a Reidemeister theory for alternating knots/tangles that stays within the realm of alternating knots/tangles?

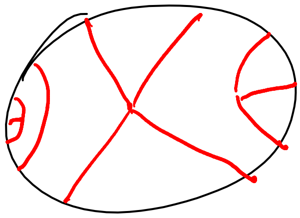
Moral: Perhaps the skein-valued tangle invariant should depend on assumptions on the connectivity of the exterior of the tangle; or perhaps the connectivity of the interior should not be forgotten.

Q: what are you worth?

A: Who's asking?

what is the planar algebra of "dual exterior connectivities"?

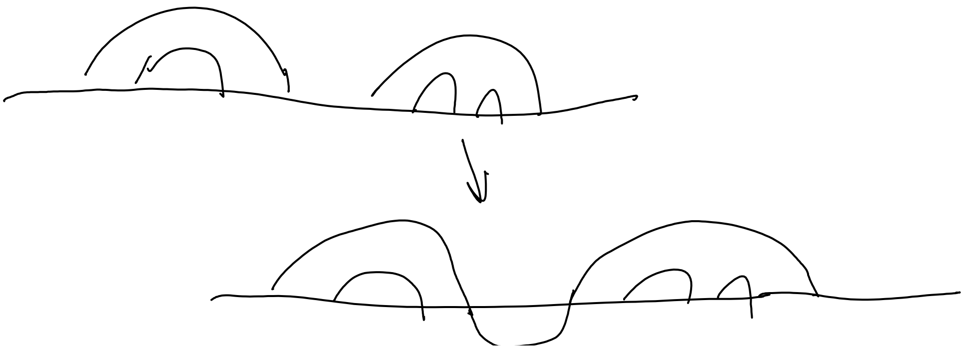
After inversion:

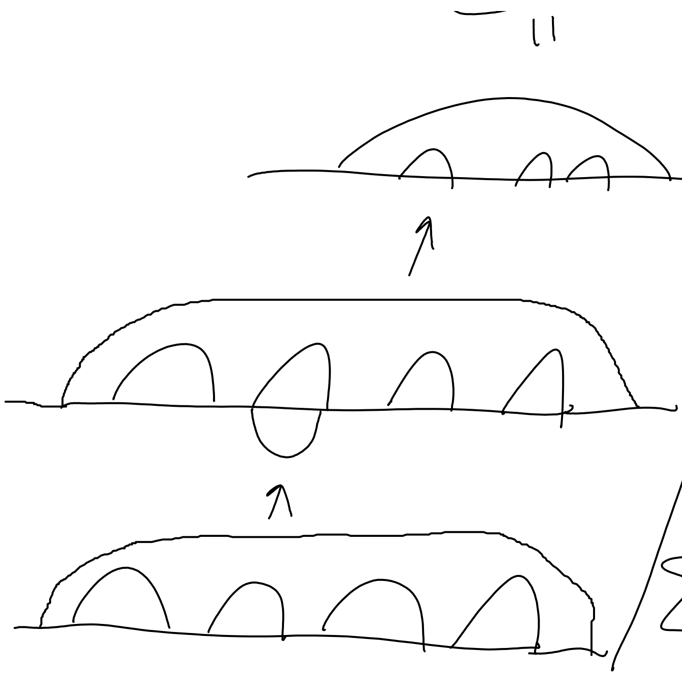


can this be described from the outside?

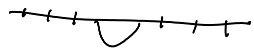
Really, we should be talking about the planar algebra of pairs:

(interior connectivity, functional on exterior connectivity)

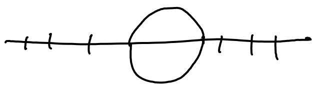




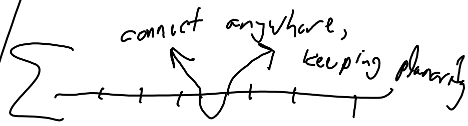
In general, the adjoint of



is



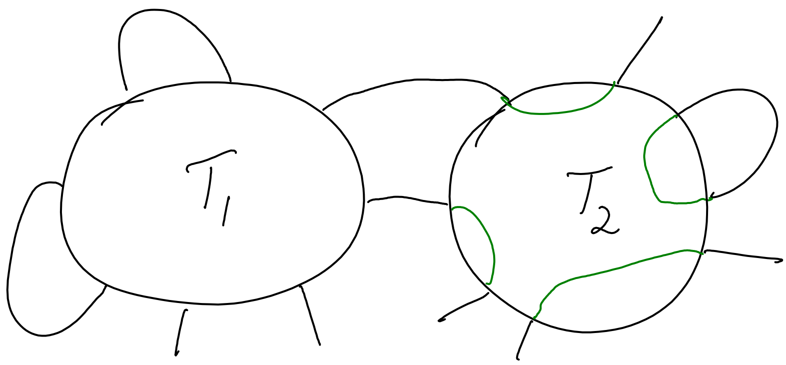
$+$



$$(- \quad -)\begin{pmatrix} q-q^{-1} & 1 \\ 1 & q-q^{-1} \end{pmatrix}\begin{pmatrix} - \\ - \end{pmatrix}$$

If T is alternating and connected, and $\overline{T}$ is any closure of T, possibly partial, then $\langle \overline{T} \rangle$ is sign alternating.**

Question Is this property stable under connected planar algebra compositions?



is this sign alternating?

* "sign alternating" for a complete closure means

* "sign alternating" for a complete closure means "after division by (A^2+A^{-2})".
* Proper care of white-yellow regions is also necessary.

Note: It is enough to consider "all-across" compositions: