Pensieve header: My first "FastKh" Khovanov homology program, retrieved from KnotTheory`.

```
<< KnotTheory`
```

Loading KnotTheory` version of February 5, 2013, 3:48:46.4762.
Read more at http://katlas.org/wiki/KnotTheory.

```
bdot[_]^_ ^= 0; tdot[_]^_ ^= 0;

EquivalenceClasses[l_List] := Module[{pos}, Fold[
    (pos = First /@ Position[#1, #2];
      Append[Delete[#1, List /@ pos], Union @@ (#1[[pos]])]) &,
    l, Union @@ l
  ]];

DotRule[top_, bot_] := DotRule[top, bot] = Module[{z}, Flatten[Cases[
      DeleteCases[
        EquivalenceClasses[Join[
          Cases[{top}, P[i_, j_][m_] :> {z@i, z@j, tdot@m}, ∞],
          Cases[{bot}, P[i_, j_][m_] :> {z@i, z@j, bdot@m}, ∞]
        ]],
        _z, {2}
      ],
      l_List :> ((# → First[l]) & /@ l)
    ]]];

HCLaw[
    Cobordism[top1_Smoothing, bot1_Smoothing],
    Cobordism[top2_Smoothing, bot2_Smoothing]
  ] /; MemberQ[{top1, bot1, top2, bot2}, Q, Infinity] := MapAt[
  (Q^Exponent[Times @@ bot1, Q] * Q^Exponent[Times @@ bot2, Q]) &,
    MapAt[
      (Q^Exponent[Times @@ top1, Q] * Q^Exponent[Times @@ top2, Q]) &,
      HCLaw[Cobordism[top1, bot1] /. Q -> 1,
        Cobordism[top2, bot2] /. Q -> 1],
      {1, 1, 1}
    ],
    {1, 2, 1}
  ];

(*
  Note: Gluing d disks along z zippers, the result has b boundaries and
  genus g with 2g=2+z-d-b.
*)
HCLaw[
 Cobordism[top1_Smoothing, bot1_Smoothing],
 Cobordism[top2_Smoothing, bot2_Smoothing]
] /; FreeQ[{top1, bot1, top2, bot2}, Q] := HCLaw[
 Cobordism[top1, bot1], Cobordism[top2, bot2]
] = Module[
 {dr, top, bot, dots, handles = 1, h, g2, decors, law, to, cob},
 dr = DotRule[top1 top2, bot1 bot2];
```

```
  top = Smoothing[
   First@top1 * First@top2 //. P[i_, j_][m_] P[j_, k_][n_] :> (
    P[i, k][Min[m, n]]
   ) /. {
    P[i_, j_][m_]^2 :> (handles /= (tdot[m] /. dr /. bdot -> h); Loop[m]),
    P[i_, i_][m_] :> (handles /= (tdot[m] /. dr /. bdot -> h); Loop[m])
   }
  ];
  bot = Smoothing[
   First@bot1 * First@bot2 //. P[i_, j_][m_] P[j_, k_][n_] :> (
    handles *= (bdot[m] /. dr /. bdot -> h);
    P[i, k][Min[m, n]]
   ) /. {
    P[i_, j_][m_]^2 :> (handles *= (bdot[m] /. dr /. bdot -> h); Loop[m]),
    P[i_, i_][m_] :> Loop[m]
   }
  ];
  dots = Union[
   Last /@ DotRule[top, bot],
   Cases[{top}, Loop[m_] :> tdot[m], Infinity],
   Cases[{bot}, Loop[m_] :> bdot[m], Infinity]
  ];
  handles *= Times @@ (Union[Last /@ dr] /. bdot -> h)^2;
  handles /= Times @@ (
   Join[
    Union[Last /@ DotRule[top1, bot1]],
    Union[Last /@ DotRule[top2, bot2]],
    Union[Last /@ DotRule[top, bot]]
   ] /. dr /. bdot -> h
  );
  decors = Expand[(handles /. h[m_]^g2_ :> (2 bdot[m])^(g2 / 2)) *
   Times @@ MapThread[
    If[#1 === #2, 1, #1 + #2] &,
    {dots, dots /. dr}
   ]
  ];
  law = Union[
   Last /@ DotRule[top1, bot1], Last /@ DotRule[top2, bot2]
  ];
  law = DeleteCases[
   Thread[to[law, law /. dr]],
   to[m_, m_]
  ] /. to -> Rule;
  {Cobordism[top, bot, decors], law}
 ];

HC[0, _] = HC[_, 0] = 0;
HC[Smoothing[s1_], Smoothing[s2_]] := Smoothing[
 s1 s2 //. P[i_, j_][m_] P[j_, k_][n_] :> P[i, k][Min[m, n]]
```

```mathematica
     /. {P[i_, j_][m_]^2 :> Loop[m], P[i_, i_][m_] :> Loop[m]}
];

HC[n1_. * e[t1__] * s1_Smoothing, n2_. * e[t2__] * s2_Smoothing] :=
  n1 n2 e[t1, t2] HC[s1, s2];

HC[
   Cobordism[top1_Smoothing, bot1_Smoothing, ds1_],
   Cobordism[top2_Smoothing, bot2_Smoothing, ds2_]
   ] := Module[
   {cob, law},
   {cob, law} = HCLaw[
     Cobordism[top1, bot1], Cobordism[top2, bot2]
     ];
   cob = MapAt[Expand[(ds1 ds2 /. law) * #] &, cob, 3];
   cob
   ];

HC[a_Plus, b_] := HC[#, b] & /@ a;
HC[a_, b_Plus] := HC[a, #] & /@ b;

HC[Morphism[top_, bot_, a_ + b_], s_] := Plus[
   HC[Morphism[top, bot, a], s],
   HC[Morphism[top, bot, b], s]
   ];
HC[Morphism[top_, bot_, MM[e[i___], e[j___], mat_]], e[k___] * s_Smoothing] :=
  Module[
   {cob, law},
   {cob, law} = HCLaw[
     Cobordism[Coefficient[top, e[i]], Coefficient[bot, e[j]]],
     Cobordism[s, s]
     ];
   MM[e[i, k], e[j, k], Expand[Last[cob] * (mat /. law)]]
   ];

HC[s_, Morphism[top_, bot_, a_Plus]] := HC[s, Morphism[top, bot, #]] & /@ a

HC[e[k___] * s_Smoothing, Morphism[top_, bot_, MM[e[i___], e[j___], mat_]]] :=
  Module[
   {cob, law},
   {cob, law} = HCLaw[
     Cobordism[s, s],
     Cobordism[Coefficient[top, e[i]], Coefficient[bot, e[j]]]
     ];
   MM[e[k, i], e[k, j], Expand[Last[cob] * (mat /. law)]]
   ];

HC[
   Kom[f1_, obs1_, mos1_],
```

```
Kom[f2_, obs2_, mos2_]
] := Module[
{l1, l2, k, j1, j2, obs, morph, mos, rule},
l1 = Length[obs1] - 1; l2 = Length[obs2] - 1;
obs = Objects @@ Table[
   Plus @@ Table[
     j2 = k - j1;
     HC[obs1[[1 + j1]], obs2[[1 + j2]]] /.
      e[t__] :> e[t, j1],
     {j1, Max[0, k - l2], Min[l1, k]}
     ],
    {k, 0, l1 + l2}
    ];
mos = Morphisms @@ Table[
   Plus @@ Table[
     j2 = k - j1;
     Plus[
      If[1 + j1 > l1 || mos1[[1 + j1]] === 0 || obs2[[1 + j2]] === 0,
       0,
       HC[
         Morphism[obs1[[1 + j1]], obs1[[2 + j1]], mos1[[1 + j1]]],
         obs2[[1 + j2]]
         ] /.
        MM[e[t1__], e[t2__], mm_] :>
         MM[e[t1, j1], e[t2, j1 + 1], mm]
       ],
      If[1 + j2 > l2 || obs1[[1 + j1]] === 0 || mos2[[1 + j2]] === 0,
       0,
       HC[
         obs1[[1 + j1]],
         Morphism[obs2[[1 + j2]], obs2[[2 + j2]], mos2[[1 + j2]]]
         ] /.
        MM[e[t1__], e[t2__], mm_] :>
         MM[e[t1, j1], e[t2, j1], Expand[(-1)^j1 * mm]]
       ]
      ],
     {j1, Max[0, k - l2], Min[l1, k]}
     ],
    {k, 0, l1 + l2 - 1}
    ];
ReTag[Kom[f1 + f2, obs, mos]]
];
```

```
ReTag[kom_Kom] := Module[
  {f, obs, mos, l},
  {f, obs, mos} = List @@ kom;
  l = Length[obs] - 1;
  Do[
   rule = Union[Cases[{obs[[1 + k]]}, _e, Infinity]];
   rule = Thread[Rule[rule, e /@ Range[Length[rule]]]];
   obs[[1 + k]] = obs[[1 + k]] /. rule;
   If[k < l,
    mos[[1 + k]] =
     mos[[1 + k]] /.
      MM[e1_, e2_, mm_] :> MM[e1 /. rule, e2, mm]
    ];
   If[k > 0,
    mos[[k]] =
     mos[[k]] /. MM[e1_, e2_, mm_] :> MM[e1, e2 /. rule, mm]
    ],
   {k, 0, l}
   ];
  Kom[f, obs, mos]
  ]
```

```mathematica
VCLaw[Cobordism[top_Smoothing, mid_Smoothing],
   Cobordism[mid_Smoothing, bot_Smoothing]] :=
 VCLaw[Cobordism[top, mid], Cobordism[mid, bot]] = Module[
   {decors, law1 = {}, law2 = {}, dots, dots1, dots2, dr1, dr2, dr, to, h, g2},
   decors = Times @@ Cases[{mid},
      Loop[m_] :> (AppendTo[law1, bdot[m] -> mdot[m]];
        AppendTo[law2, tdot[m] → mdot[m]]; mdot[m]),
      Infinity];
   dots = Union[Last /@ DotRule[top, bot]];
   dots1 = Union[Last /@ (dr1 = DotRule[top, mid] /. bdot → mdot)];
   dots2 = Union[Last /@ (dr2 = DotRule[mid, bot] /. tdot → mdot)];
   dr = Flatten[Cases[EquivalenceClasses[Join[List @@@ dr1, List @@@ dr2]],
      l_List :> ((# → First[l]) & /@ Rest[l]) ]];
   decors *= Times @@ (Union[Last /@ dr] /. bdot → h)^2;
   decors *=
    Times @@ (Cases[mid, P[__][m_] :> mdot[m], Infinity] /. dr /. bdot → h);
   decors /= Times @@ (Join[dots1, dots2, dots] /. dr /. bdot → h);
   decors = decors /. h[m_]^g2_ :> (2 bdot[m])^(g2 / 2);
   decors *= Expand[Times @@ MapThread[If[#1 === #2, 1, #1 + #2] &,
      {dots, dots /. dr}]];
   law1 = Join[law1,
     DeleteCases[
       Thread[to[dots1, dots1 /. dr]] /. mdot -> bdot,
       to[m_, m_]
      ] /. to -> Rule];
   law2 = Join[law2,
     DeleteCases[
       Thread[to[dots2, dots2 /. dr]],
       to[m_, m_]
      ] /. to -> Rule];
   {law1, law2, decors}
  ];

VC[a_, b_, c__] := VC[a, VC[b, c]];
VC[Cobordism[top_Smoothing, mid_Smoothing, ds1_],
   Cobordism[mid_Smoothing, bot_Smoothing, ds2_]] := Module[
  {law1, law2, decor},
  {law1, law2, decor} = VCLaw[Cobordism[top, mid], Cobordism[mid, bot]];
  Cobordism[top, bot,

   Expand[decor * (ds1 /. law1) * (ds2 /. law2)] /. (_mdot)^2 → 1 /. (_mdot -> 0)]
  ];
```

```mathematica
DeLoop[kom_Kom] := Module[
   {f, obs, mos, l, dot},
   {f, obs, mos} = List @@ kom;
   l = Length[obs] - 1;
   Do[
    obs[[1 + k]] =
     obs[[1 + k]] //. e[i___] Smoothing[Loop[j_] * rest_.] :> (
        If[k > 0,
         mos[[k]] =
          mos[[k]] /. MM[e[l___], e[i], mat_] :> Plus[
             MM[e[l], e[i, -1],
              Expand[dot[j] * mat] /. bdot[j] dot[j] -> 1 /.
               dot[j] -> 0
              ],
             MM[e[l], e[i, 1],
              mat /. bdot[j] -> 0
              ]
            ]];
        If[k < l,
         mos[[1 + k]] =
          mos[[1 + k]] /.
           MM[e[i], e[l___], mat_] :> Plus[
             MM[e[i, -1], e[l],
              mat /. tdot[j] -> 0
              ],
             MM[e[i, 1], e[l],
              Expand[dot[j] * mat] /. tdot[j] dot[j] -> 1 /.
               dot[j] -> 0
              ]
            ]];
        e[i, -1] Smoothing[rest / Q] + e[i, 1] Smoothing[rest * Q]
        ),
    {k, 0, l}
    ];
   ReTag[Kom[f, obs, mos] /. MM[_, _, {{0}}] -> 0]
   ];

Contract[kom_Kom] := Module[
   {f, obs, mos, l, k, e2s0, e2s1,
     s2b, b, e2b0, e2b1, killed0, killed1, done, mok},
   {f, obs, mos} = List @@ kom;
   l = Length[obs] - 1;
   Do[ (* {k,0,l-1} *)
     e2s0 = Cases[{obs[[1 + k]]}, i_e * s_Smoothing :> (i -> s), Infinity];
     e2s1 = Cases[{obs[[1 + k + 1]]}, i_e * s_Smoothing :> (i -> s), Infinity];
     s2b = Union[Union[Last /@ e2s0, Last /@ e2s1] /. P[j__][m_] :> P[j]];
     s2b = Thread[Rule[s2b, b /@ Range[Length[s2b]]]];
     e2b0 = e2s0 /. P[j__][m_] :> P[j] /. s2b;
     e2b1 = e2s1 /. P[j__][m_] :> P[j] /. s2b;
```

```
     killed0 = killed1 = {}; done = False;
     While[!done,
      done = True;
      mok = mos[[1 + k]];
     Cases[{mok},
        MM[i_e, j_e, {{r_?NumberQ}}] /; ((i /. e2b0) === (j /. e2b1)) :> (
          mok = Plus[
            mok /. {MM[i, _, _] -> 0, MM[_, j, _] -> 0},
             Expand[-Plus @@ Flatten[Outer[
                   Function[{M1, M2},
                     MM[M1[[1]], M2[[2]], Last[VC[
                         Cordism[M1[[1]] /. e2s0, j /. e2s1, M1[[3, 1, 1]]],
                         Cordism[j /. e2s1, i /. e2s0, {{1 / r}}],
                         Cordism[i /. e2s0, M2[[2]] /. e2s1, M2[[3, 1, 1]]]
                       ]]
                     ]
                   ],
                   Cases[{mok}, MM[i1_e, j, mm1_] /; i1 =!= i, Infinity],
                   Cases[{mok}, MM[i, j1_e, mm2_] /; j1 =!= j, Infinity]
                 ]]]
            ];
          mos[[1 + k]] =
            (((mok //. a_ * MM[i1_, j1_, mm_] :> MM[i1, j1, Expand[a * mm]]) //.
                MM[i1_, j1_, mm1_] + MM[i1_, j1_, mm2_] :>
                 MM[i1, j1, mm1 + mm2]) /.
              MM[_, _, {{0}}] -> 0);
          done = False;
          AppendTo[killed0, i]; AppendTo[killed1, j]
          ),
        Infinity, 1]
      ];
     obs[[1 + k]] = obs[[1 + k]] /. ((# -> 0) & /@ killed0);
     obs[[1 + k + 1]] = obs[[1 + k + 1]] /. ((# -> 0) & /@ killed1);
     If[k > 0, mos[[1 + k - 1]] =
        mos[[1 + k - 1]] /. MM[i_e, j_e, mm_] /; MemberQ[killed0, j] :> 0];
     If[k < l - 1, mos[[1 + k + 1]] = mos[[1 + k + 1]] /.
          MM[i_e, j_e, mm_] /; MemberQ[killed1, i] :> 0],
     {k, 0, l - 1}
    ];
   ReTag[Kom[f, obs, mos]]
   ];

KhComplex[X[i_, j_, k_, l_]] /; (j - l == 1 || l - j > 1) := Kom[0, (* + xing *)
  Objects[
    e[1] Smoothing[Q P[i, j] P[k, l]],
    e[1] Smoothing[Q^2 P[i, l] P[j, k]]
    ] /. P[m_, n_] :> P[m, n][Min[m, n]],
  Morphisms[MM[e[1], e[1], {{1}}]]]
  ];
```

```mathematica
KhComplex[X[i_, j_, k_, l_]] /; (l - j == 1 || j - l > 1) := Kom[-1, (* - xing *)
   Objects[
     e[1] Smoothing[Q^(-2) P[i, j] P[k, l]],
     e[1] Smoothing[Q^(-1) P[i, l] P[j, k]]
     ] /. P[m_, n_] :> P[m, n][Min[m, n]],
   Morphisms[MM[e[1], e[1], {{1}}]]
   ];
KhComplex[pd_PD] /; (Length[pd] > 1) := Module[
  {kom},
  kom = KhComplex[First@pd];
  Do[
   kom = HC[kom, KhComplex[pd[[i]]]];
   kom = DeLoop[kom];
   kom = Contract[kom],
   {i, 2, Length[pd]}
   ];
  kom
  ]

KhPoly[kom_Kom] := Module[
   {f, obs, mos},
   {f, obs, mos} = List @@ kom;
   If[Union[List @@ mos] =!= {0}, Error,
    Plus @@ Expand[t^(f - 1) * t^Range[Length[obs]] * (
       List @@ obs /. e[i_] Smoothing[s_] :> s /. Q -> q
       )]
    ]
   ];

FastKh[L_, opts___] := FastKh[L, opts] = Module[
 {
  L1, pos, inside, L2, f, cl,
  eo = (ExpansionOrder /. {opts} /. Options[Kh])
 },
 L1 = PD[L];
 If[eo === Automatic,
   L2 = List @@ L1; L1 = PD[]; inside = {};
   While[Length[L2] > 0,
    pos = Last[Ordering[(Length[Intersection[List @@ #, inside]]) & /@ L2]];
    AppendTo[L1, L2[[pos]]];
    inside = Union[inside, List @@ L2[[pos]]];
    L2 = Delete[L2, pos]
   ]
 ];
 Function @@ {KhPoly[KhComplex[L1]] /. {q -> #1, t -> #2}}
]
```

```
TabularKh[kh_] := TabularKh[kh, {}]
TabularKh[khG_, highlight_List] :=
 Module[{kh, out, width, minr, maxr, minj, maxj, j, r, c, critical, chi},
  kh = khG /. {Global`t -> t, Global`q -> q};
  minr = Exponent[kh, t, Min];
  maxr = Exponent[kh, t, Max];
  minj = Exponent[kh, q, Min];
  maxj = Exponent[kh, q, Max];
  width = N[100 / (maxr - minr + 5)];
  out = StringJoin["<table border=1>\n", "<tr align=center>\n",
    "<td width=" <> ToString[2 width] <>
     "%><table cellpadding=0 cellspacing=0>\n",
    "  <tr><td>\\</td><td> </td><td>r</td></tr>\n",
    "<tr><td> </td><td> \\ </td><td> </td></tr>\n",
    "<tr><td>j</td><td> </td><td>\\</td></tr>\n", "</table></td>\n"];
  Do[out = out <> "<td width=" <> ToString[width] <> "%>" <> ToString[r] <> "</td>",
    {r, minr, maxr}];
  out = out <> "<td width=" <> ToString[2 width] <> "%>&chi;</td></tr>\n";
  Do[out = out <> "<tr align=center><td>" <> ToString[j] <> "</td>";
   chi = 0;
   Do[
    c = Coefficient[
       Expand[kh * t^(1 - minr) * q^(1 - minj)], t^(r + 1 - minr) * q^(j + 1 - minj)];
    chi += (-1)^r * c;
    critical = MemberQ[highlight, j - 2 r];
    out =
     out <> Which[critical && c != 0,
       "<td bgcolor=yellow>" <> ToString[c] <> "</td>",
       critical && c == 0,
       "<td bgcolor=yellow> </td>", ! critical && c != 0,
       "<td bgcolor=red>" <> ToString[c] <> "</td>", ! critical && c == 0,
       "<td> </td>"], {r, minr, maxr}];
   out = out <> "<td>" <> ToString[chi] <> "</td></tr>\n", {j, maxj, minj, -2}];
  out = out <> "</table>"]

FastKh[Knot[8, 17]][q, t]
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\frac{4}{q} + 4\,q + \frac{1}{q^9\,t^4} + \frac{2}{q^7\,t^3} + \frac{1}{q^5\,t^3} + \frac{3}{q^5\,t^2} + \frac{2}{q^3\,t^2} +$$

$$\frac{3}{q^3\,t} + \frac{3}{q\,t} + 3\,q\,t + 3\,q^3\,t + 2\,q^3\,t^2 + 3\,q^5\,t^2 + q^5\,t^3 + 2\,q^7\,t^3 + q^9\,t^4$$

Kh[Knot[8, 17]][q, t]

KnotTheory::loading : Loading precomputed data in Kh4Knots`.

$$\frac{4}{q} + 4\,q + \frac{1}{q^9\,t^4} + \frac{2}{q^7\,t^3} + \frac{1}{q^5\,t^3} + \frac{3}{q^5\,t^2} + \frac{2}{q^3\,t^2} +$$

$$\frac{3}{q^3\,t} + \frac{3}{q\,t} + 3\,q\,t + 3\,q^3\,t + 2\,q^3\,t^2 + 3\,q^5\,t^2 + q^5\,t^3 + 2\,q^7\,t^3 + q^9\,t^4$$

FastKh[TorusKnot[6, 5]][q, t] // Timing

$\{64.366013,$

$q^{19} + q^{21} + q^{23}\,t^2 + q^{27}\,t^3 + q^{25}\,t^4 + q^{27}\,t^4 + q^{29}\,t^5 + q^{31}\,t^5 + q^{27}\,t^6 + q^{29}\,t^6 + q^{31}\,t^7 + q^{33}\,t^7 + q^{29}\,t^8 +$

$2\,q^{31}\,t^8 + q^{33}\,t^9 + 2\,q^{35}\,t^9 + q^{33}\,t^{10} + 2\,q^{37}\,t^{11} + q^{35}\,t^{12} + q^{37}\,t^{12} + q^{41}\,t^{12} + q^{39}\,t^{13} + q^{41}\,t^{13}\}$

Kh[PD[TorusKnot[6, 5]], Program → "FastKh"][q, t] // Timing

KnotTheory::credits : The Khovanov homology program FastKh was written by Dror Bar–Natan.

$\{71.089656,$

$q^{19} + q^{21} + q^{23}\,t^2 + q^{27}\,t^3 + q^{25}\,t^4 + q^{27}\,t^4 + q^{29}\,t^5 + q^{31}\,t^5 + q^{27}\,t^6 + q^{29}\,t^6 + q^{31}\,t^7 + q^{33}\,t^7 + q^{29}\,t^8 +$

$2\,q^{31}\,t^8 + q^{33}\,t^9 + 2\,q^{35}\,t^9 + q^{33}\,t^{10} + 2\,q^{37}\,t^{11} + q^{35}\,t^{12} + q^{37}\,t^{12} + q^{41}\,t^{12} + q^{39}\,t^{13} + q^{41}\,t^{13}\}$

FastKh[TorusKnot[9, 5]][q, t] // Timing

$\{785.777037,\ q^{31} + q^{33} + q^{35}\,t^2 + q^{39}\,t^3 + q^{37}\,t^4 + q^{39}\,t^4 + q^{41}\,t^5 + q^{43}\,t^5 + q^{39}\,t^6 + q^{41}\,t^6 + q^{43}\,t^7 +$

$q^{45}\,t^7 + q^{41}\,t^8 + 2\,q^{43}\,t^8 + q^{45}\,t^9 + 2\,q^{47}\,t^9 + 2\,q^{45}\,t^{10} + 3\,q^{49}\,t^{11} + 2\,q^{47}\,t^{12} + 2\,q^{49}\,t^{12} + q^{53}\,t^{12} +$

$3\,q^{51}\,t^{13} + 2\,q^{53}\,t^{13} + q^{49}\,t^{14} + 2\,q^{51}\,t^{14} + q^{55}\,t^{14} + 2\,q^{53}\,t^{15} + 3\,q^{55}\,t^{15} + 2\,q^{53}\,t^{16} + q^{57}\,t^{16} +$

$q^{59}\,t^{16} + 3\,q^{57}\,t^{17} + q^{55}\,t^{18} + q^{57}\,t^{18} + q^{61}\,t^{18} + 2\,q^{59}\,t^{19} + q^{61}\,t^{19} + q^{59}\,t^{20} + q^{63}\,t^{20} + q^{63}\,t^{21}\}$

Kh[PD[TorusKnot[9, 5]], Program → "FastKh"][q, t] // Timing

$\{722.768233,\ q^{31} + q^{33} + q^{35}\,t^2 + q^{39}\,t^3 + q^{37}\,t^4 + q^{39}\,t^4 + q^{41}\,t^5 + q^{43}\,t^5 + q^{39}\,t^6 + q^{41}\,t^6 + q^{43}\,t^7 +$

$q^{45}\,t^7 + q^{41}\,t^8 + 2\,q^{43}\,t^8 + q^{45}\,t^9 + 2\,q^{47}\,t^9 + 2\,q^{45}\,t^{10} + 3\,q^{49}\,t^{11} + 2\,q^{47}\,t^{12} + 2\,q^{49}\,t^{12} + q^{53}\,t^{12} +$

$3\,q^{51}\,t^{13} + 2\,q^{53}\,t^{13} + q^{49}\,t^{14} + 2\,q^{51}\,t^{14} + q^{55}\,t^{14} + 2\,q^{53}\,t^{15} + 3\,q^{55}\,t^{15} + 2\,q^{53}\,t^{16} + q^{57}\,t^{16} +$

$q^{59}\,t^{16} + 3\,q^{57}\,t^{17} + q^{55}\,t^{18} + q^{57}\,t^{18} + q^{61}\,t^{18} + 2\,q^{59}\,t^{19} + q^{61}\,t^{19} + q^{59}\,t^{20} + q^{63}\,t^{20} + q^{63}\,t^{21}\}$

FastKh[TorusKnot[7, 6]][q, t] // Timing

Kh[PD[TorusKnot[7, 6]], Program → "FastKh"][q, t] // Timing