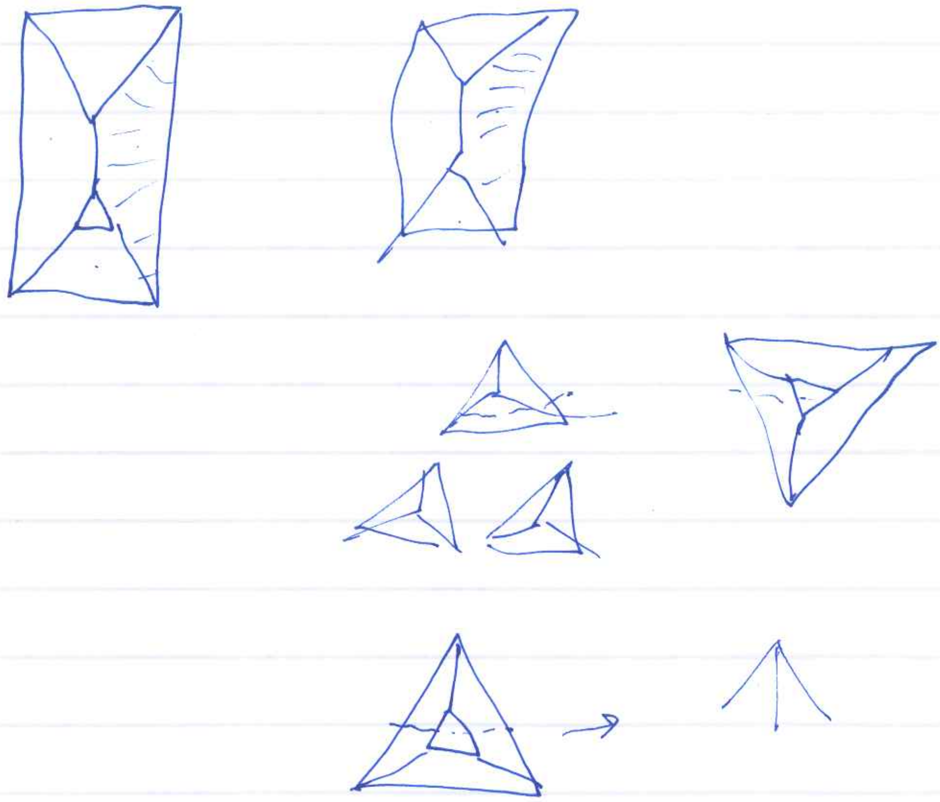
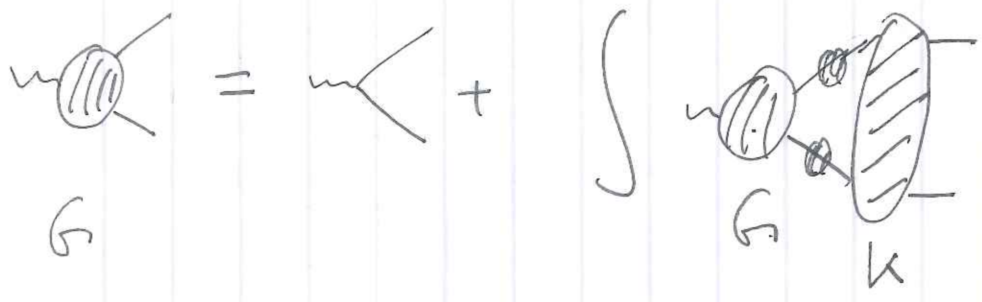
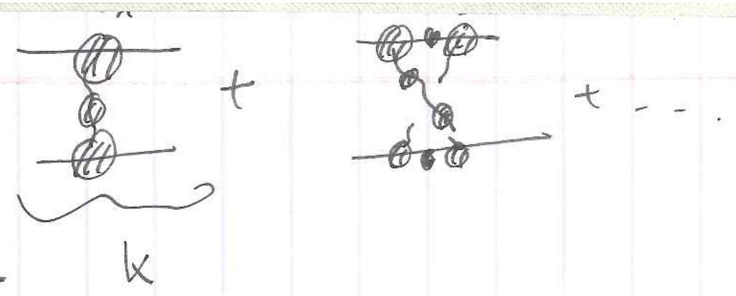


3 1. Top. failed attempt
7 2. analytical attempts } on
 * KZ. board?
 * Conf. spaces

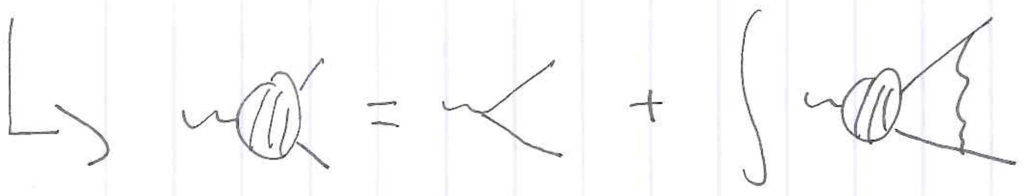
8 3. ~~[illegible]~~ KTG
 and it's ops
8 4. Generators
5 5. relations
10 6. The pentagon

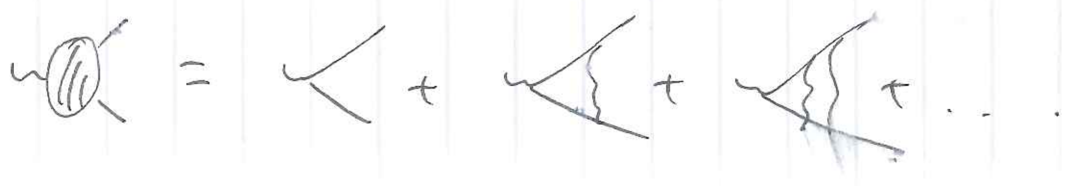
41.





Σ

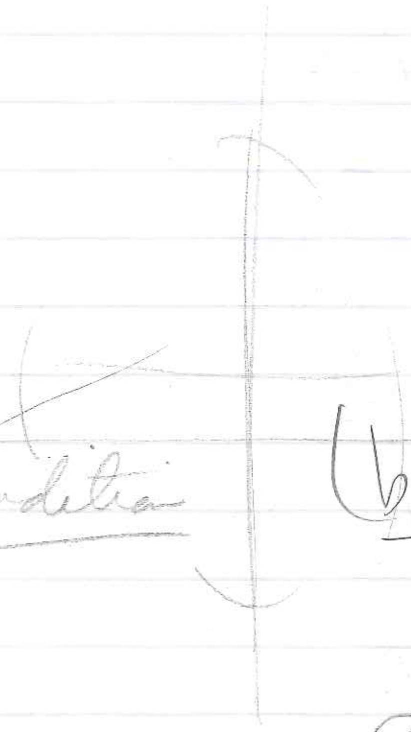




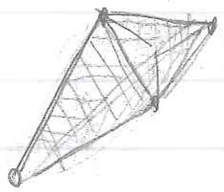
② $|X \times Y| = |X| \times |Y|$

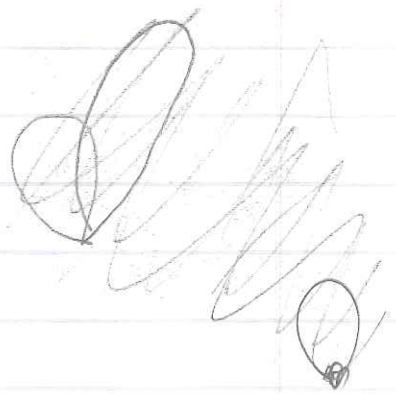
$GX \times GX \to GX$

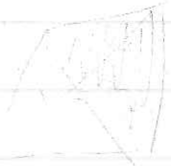
$|GX|$



④ Kon condition (bigness)

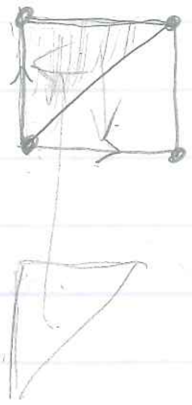




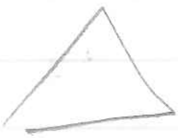


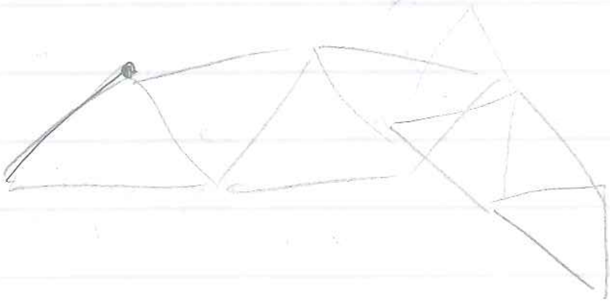
① $(X\times Y)_n = X_n \times Y_n$

$\coprod(\sigma) \big/ \partial_i\sigma \sim$



③ $(\Delta_n) \xrightarrow[\text{map}]{\text{simp}} (\text{simplicial complex})$





R^n_x

$\Delta^{n-1} \xrightarrow{n+1} \Delta^{n}$

$\Delta^{n+1} \xrightarrow{n+1} \Delta^{n}$

$z_0 \ldots z_{n+1}$

$R^{n-1}_{+0} \xrightarrow{n} R^n_+$

$R^{n+1}_{0+1} \xrightarrow{n+1} \mathbb{R}^n_+$

~~9. The planar set S~~

9. Examples.

10. The planar set S
    Fibered planar algebras
    more examples.

11. The RT theorem

12. The NoGo.

13. The planar alg ~~SU(3)~~ SU

14. multiplication

15. Theorem & question

Knot invariants, Associators & a strange breed of planar algebras. The Fields Institute, Jun 11, 2001

slides: ~~Abs~~ 1. Abstract.

2. ~~Goal~~ Mathematical Goal:

Meta mathematical goals. 1. Tell about a ~~Rimer~~ fine ~~beautiful~~ but intriguing theorem, that although multiply proven is still not properly understood

~~3. State a theorem & pose a question~~

2. Introduce "a strange breed of planar algebras", state a theorem and pose a question

3. Confess that I still don't understand associators.

3. The CS way

4. The KZ way

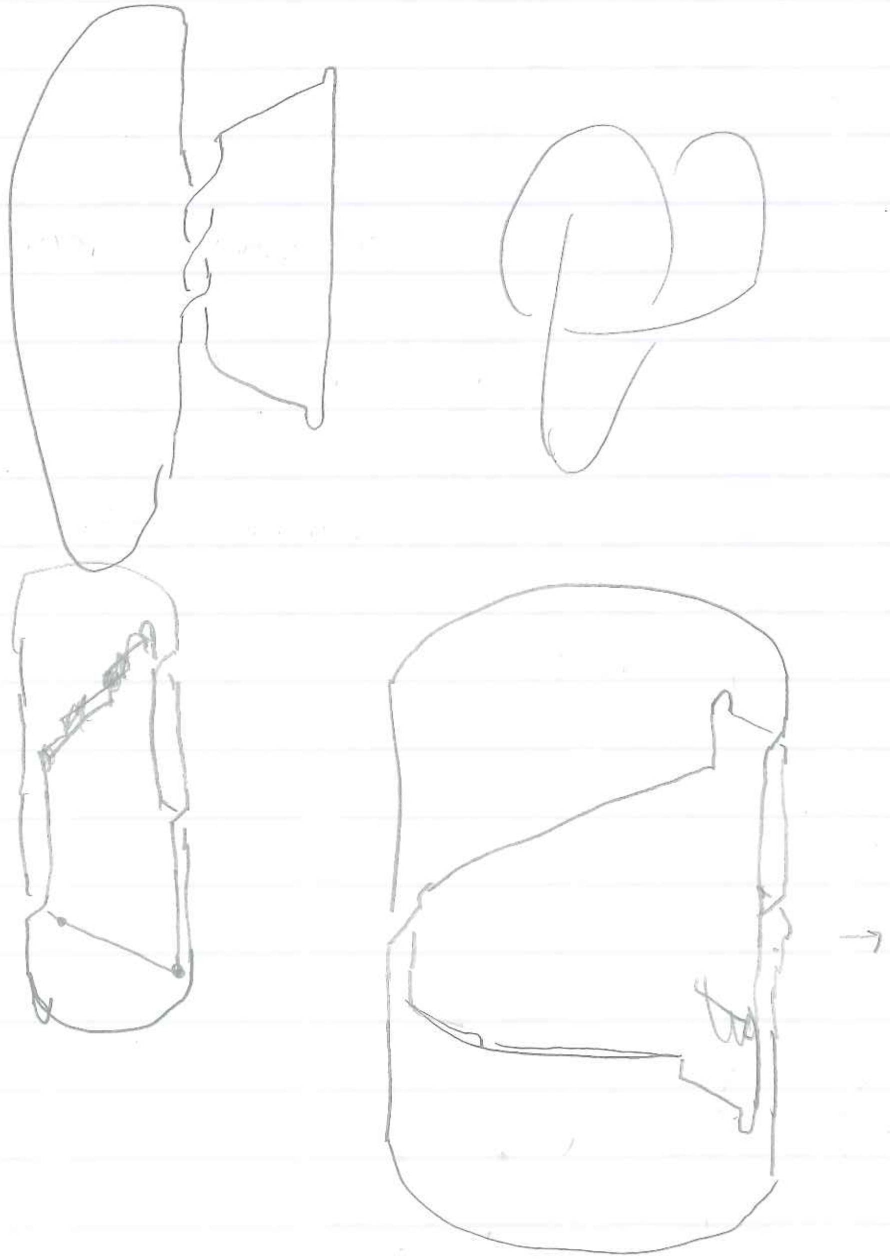
5. ~~Drinfeld's way~~ Hatching's way

6. Drinfeld's way

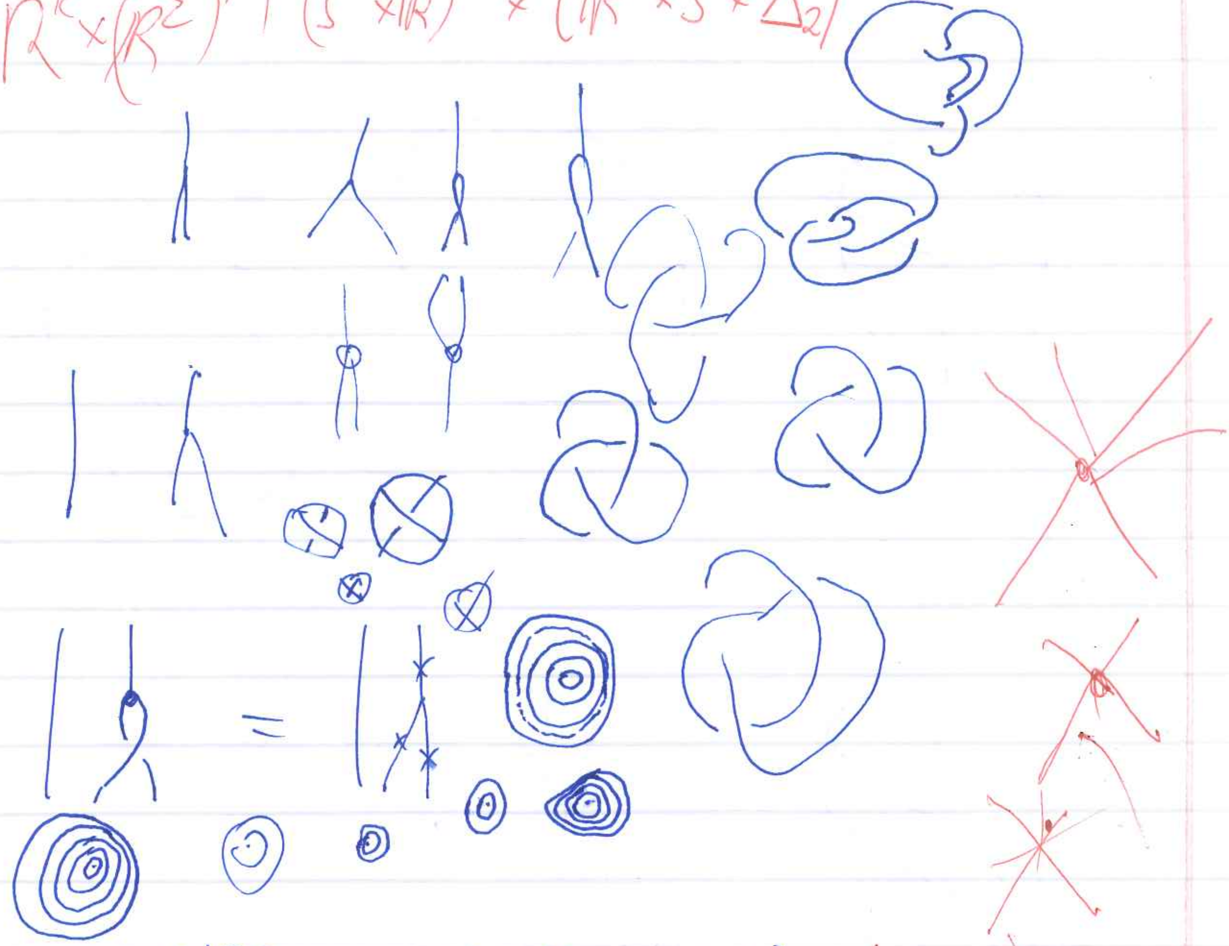
7. knotted trivalent graphs

8. Planar ~~al~~ algebras (and directed planar algs)

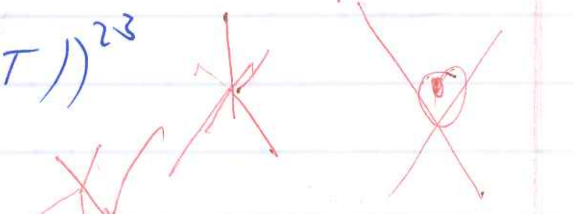
$$\Omega_{KZ} = \sum_{i \neq j} \sum_{a} 1 \otimes \cdots \otimes \overset{i}{t_a} \otimes \cdots \otimes \overset{j}{t_a} \otimes \cdots \otimes 1 \; \frac{dz_i - dz_j}{z_i - z_j}$$

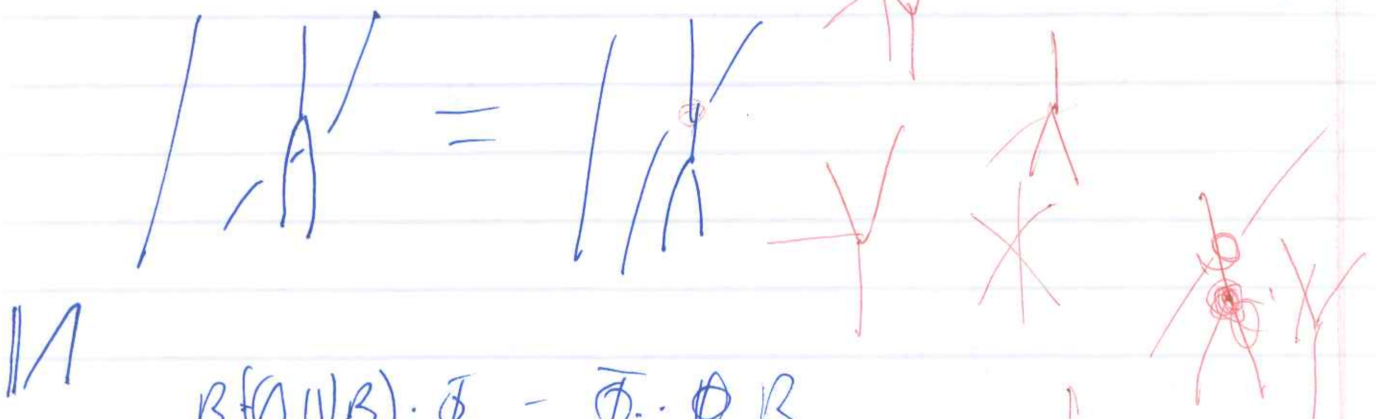


$\mathbb{R}^k \times (\mathbb{R}^2)^n \quad (S^1 \times \mathbb{R})^k \times (\mathbb{R}^2 \times S^1 \times \Delta_2)^n$

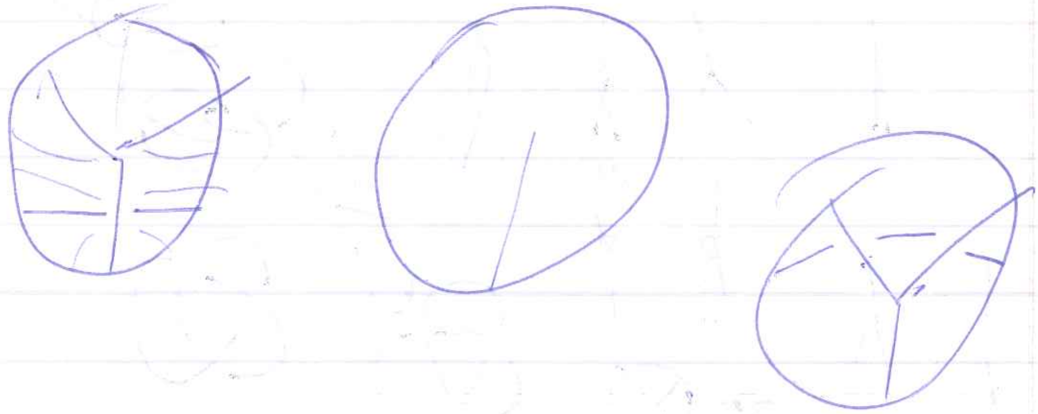


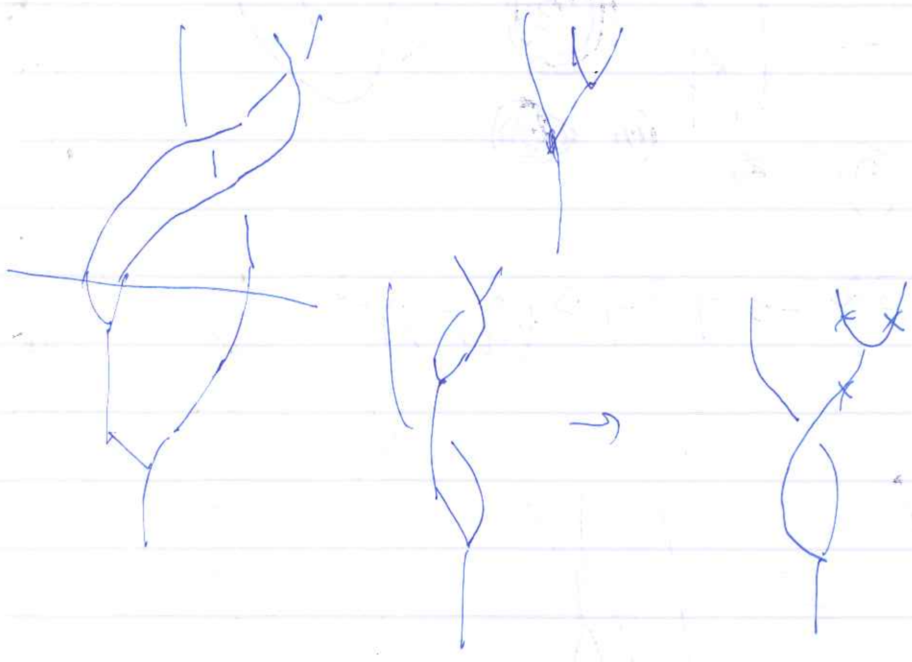
$$B\cdot\Phi^{132} = T^2 T^3 \cdot (\Delta(T))^{23}$$

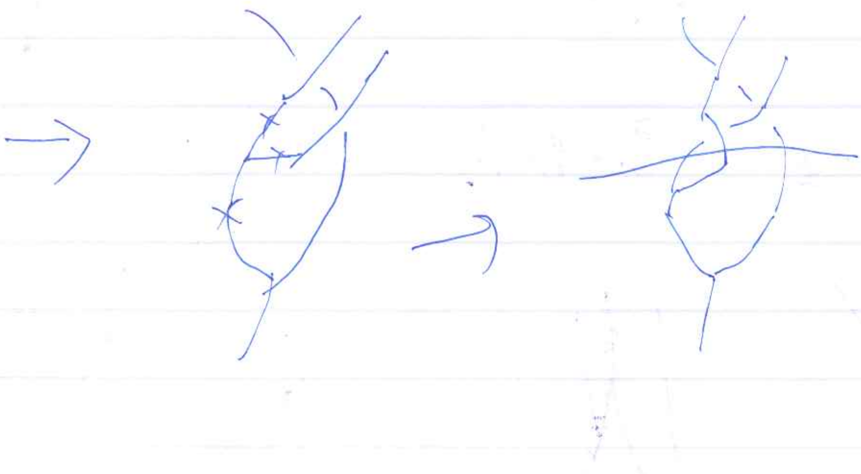


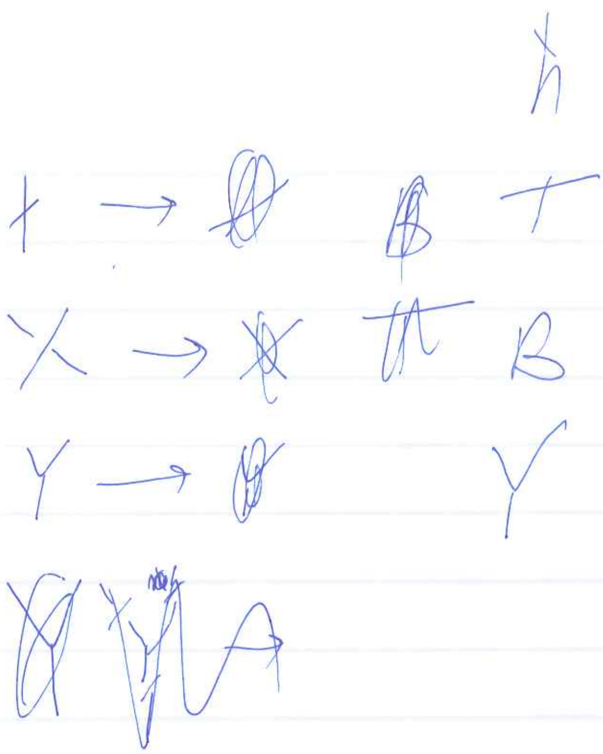


$$B(\Delta \otimes 1)B)\cdot\Phi = \Phi\cdot\Phi B$$

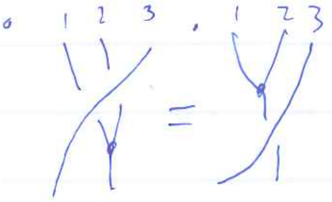








$$T^{(12)} \cdot Y = Y^{021} T^1 T^2 \cdot B$$



$$Y^{(03)12} \cdot B^{013} \cdot B^{(01)23} = B^{0(12)3} \cdot Y$$

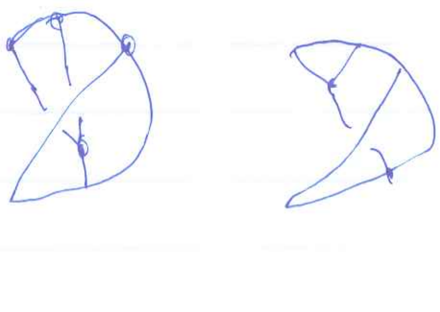
~~8Y~~

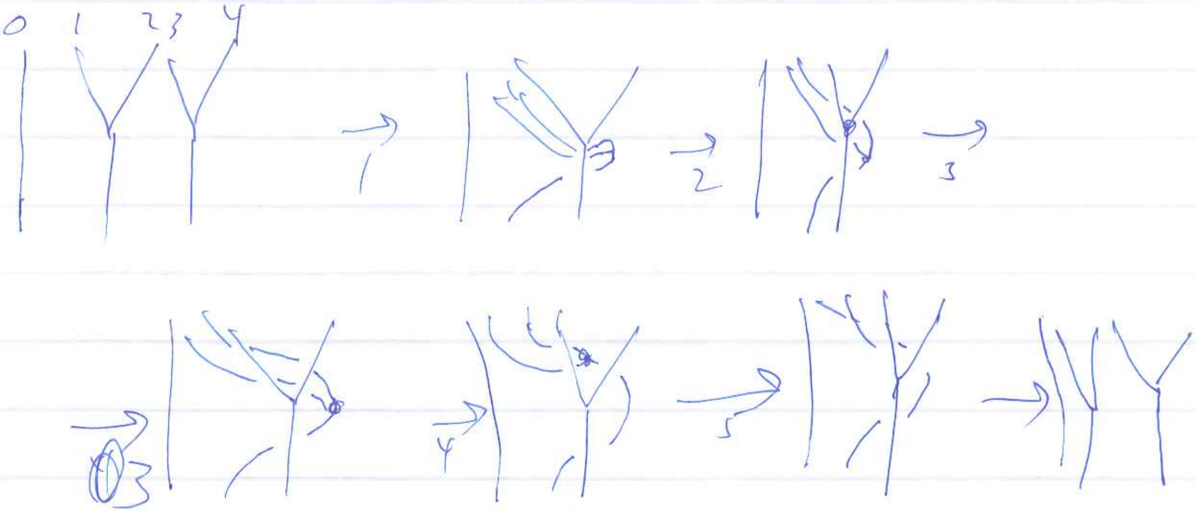
~~8B → 8Y + 16R~~

8  $6 P_5$

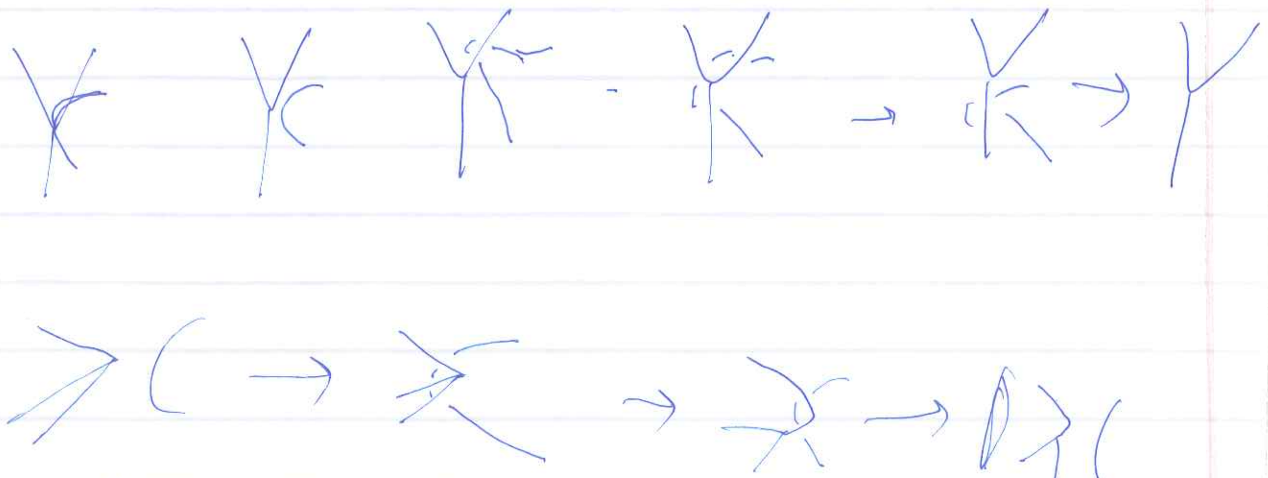
$8 R_3$

$4 P_5 + 3 R_3$





E

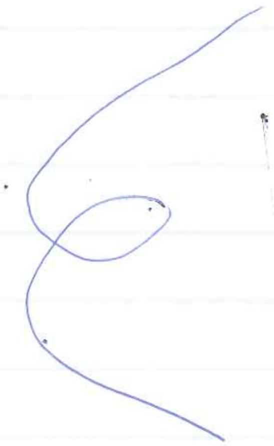


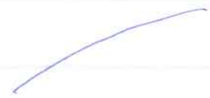
$$\gamma(t) = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} t + \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} t^2 + \begin{pmatrix} a_3 \\ b_3 \end{pmatrix} t^3$$

[illegible]

$$\gamma(t) = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} t + \ldots$$

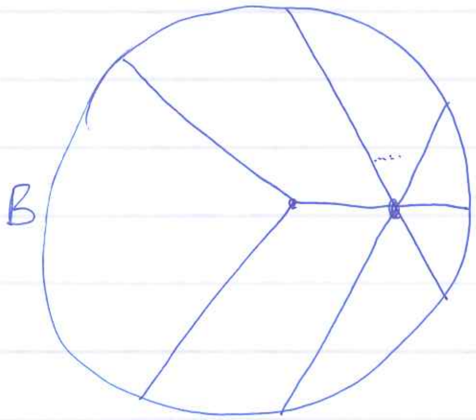
$$= \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} t + \begin{pmatrix} \alpha_2 \\ 1 \end{pmatrix} t^2 + \begin{pmatrix} a_3 \\ b_3 \end{pmatrix} t^3$$

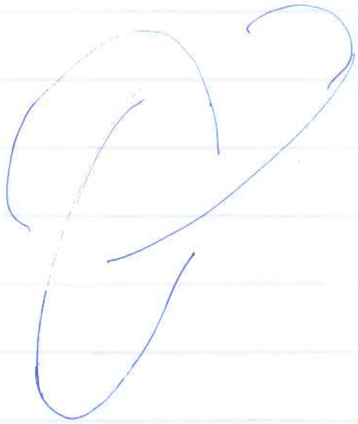


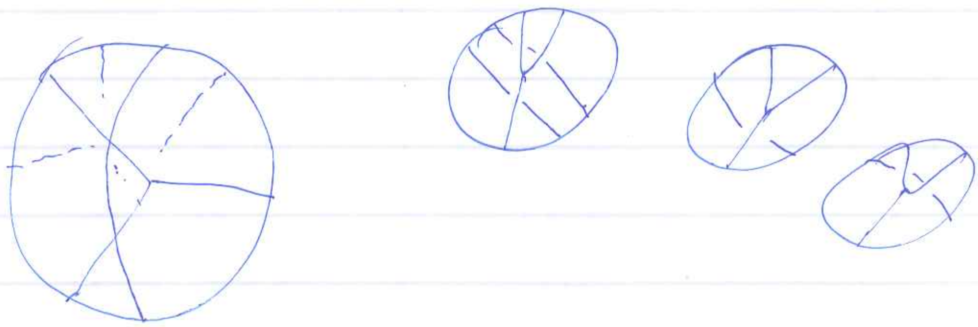


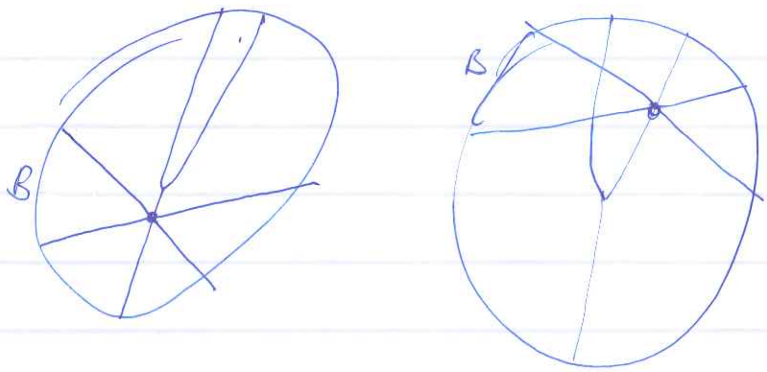
$*t + 1t^2$

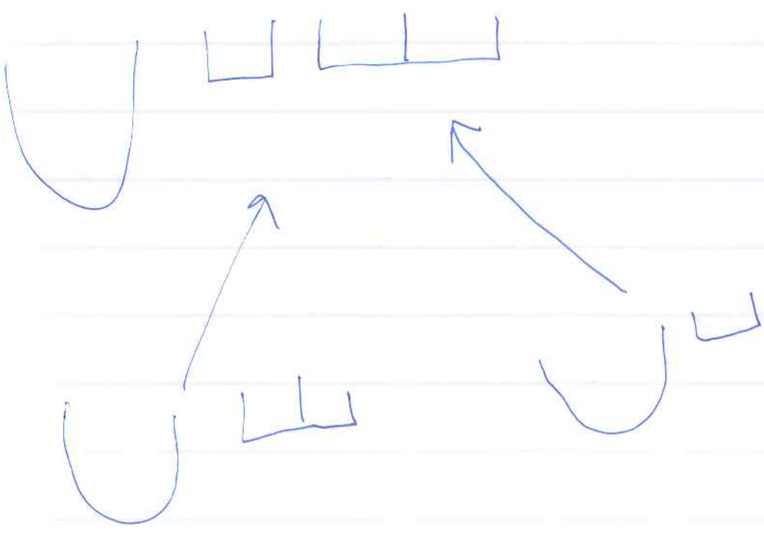
$*t + *t^2 + t^3$

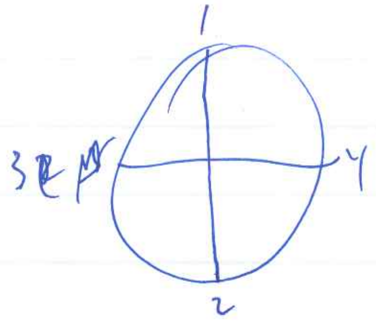

B

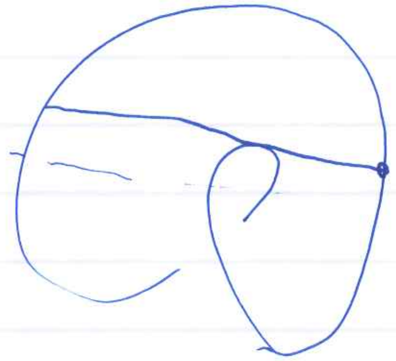


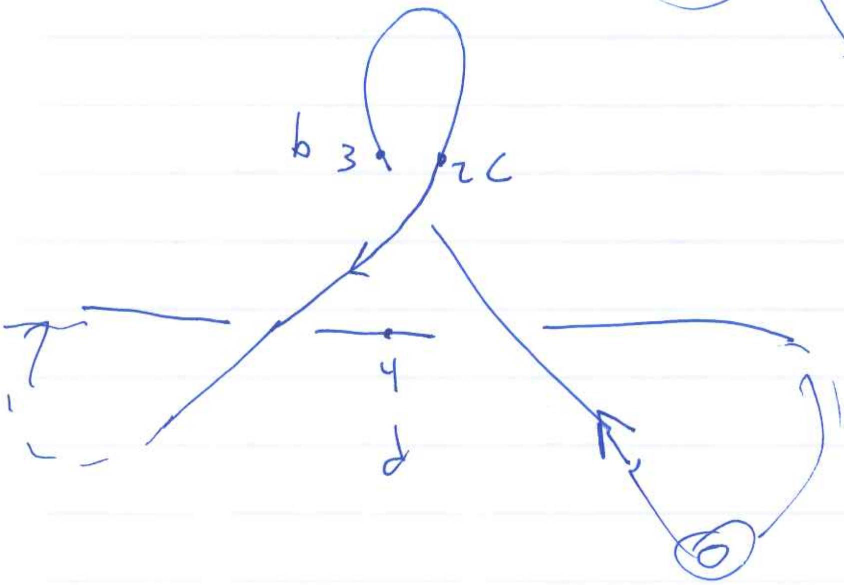


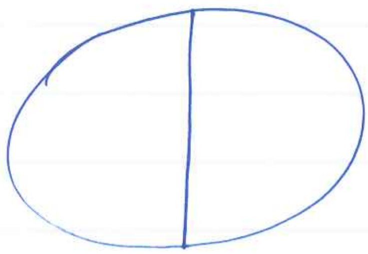

B
B

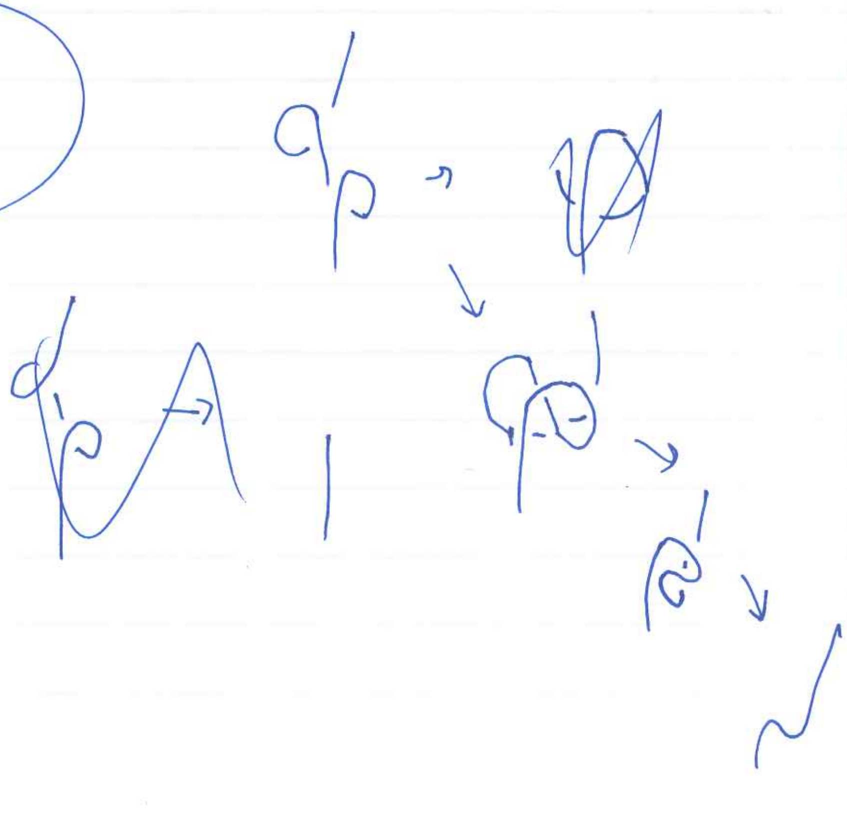


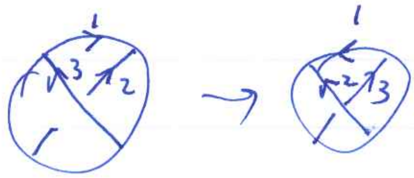

1
3
4
2

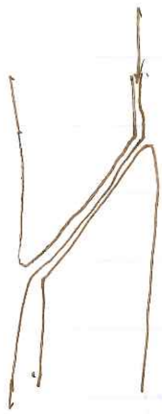


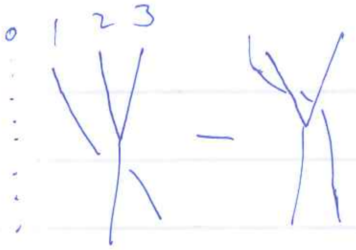

b 3
2 c
4
d



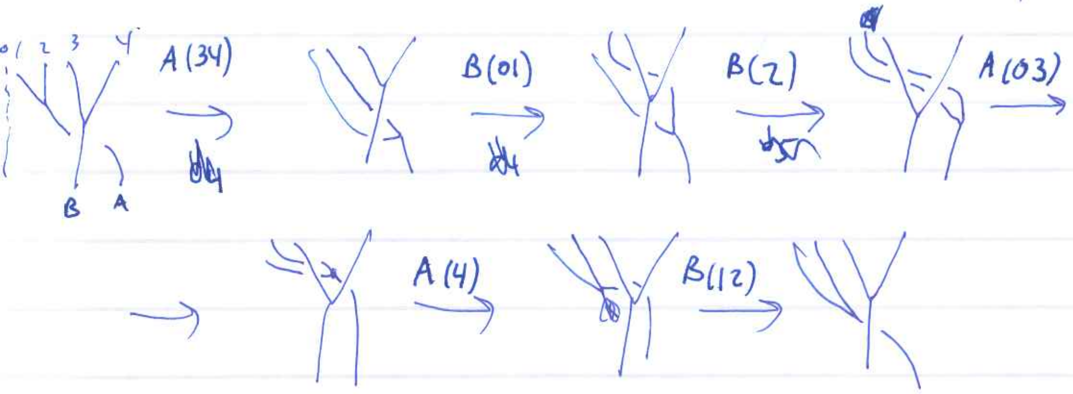



1
3
2
1
2
3





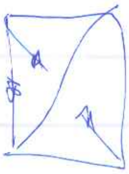
$$\Delta\Psi = \Delta\eta^{01,23} - \Delta\eta^{0,1,3} + \Delta\beta^{0,1,23} - \Delta\beta^{02,1,3} - \Delta\beta^{0,1,2}$$

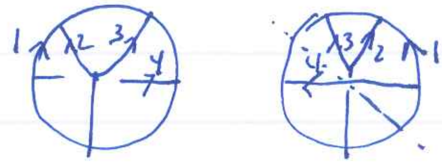


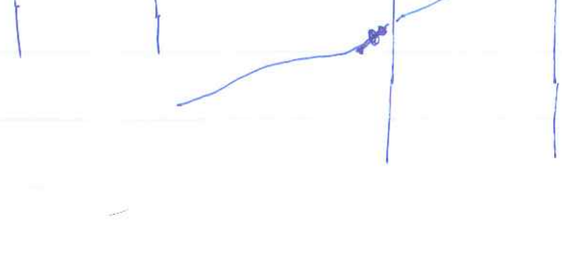
$$\gamma = -\beta^{021}$$

$$\beta = \gamma^{01,2,-1} =$$

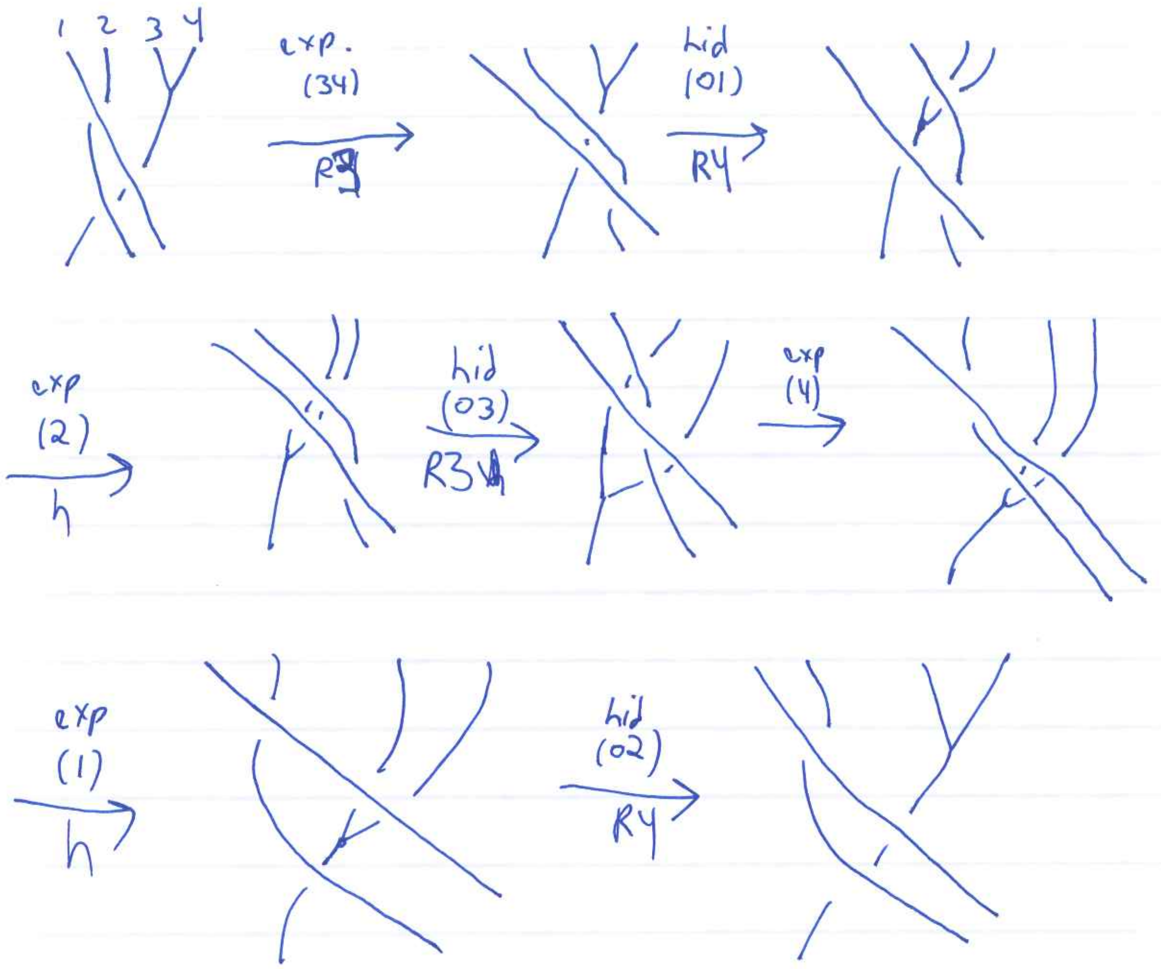
$$= -\beta^{01,-1,2}$$





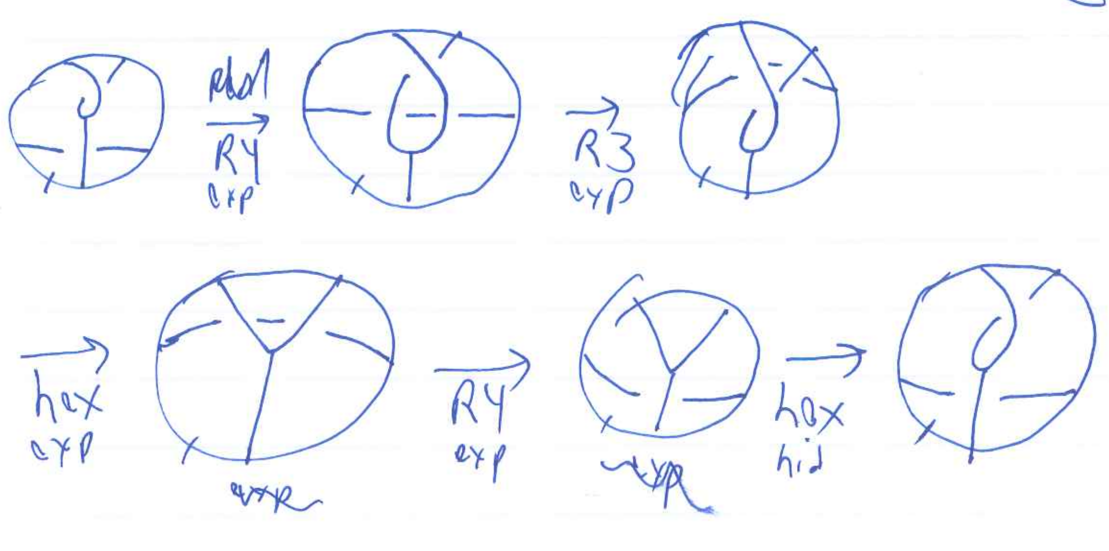


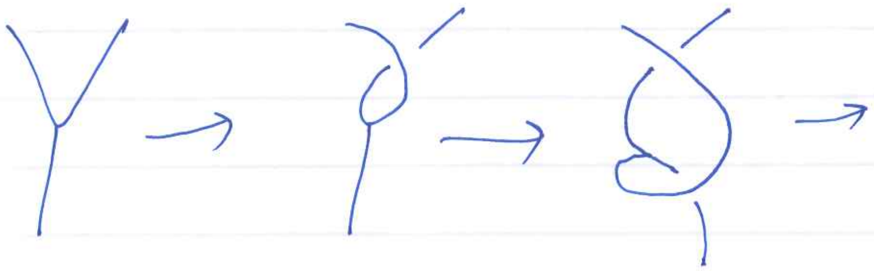
magnets website.

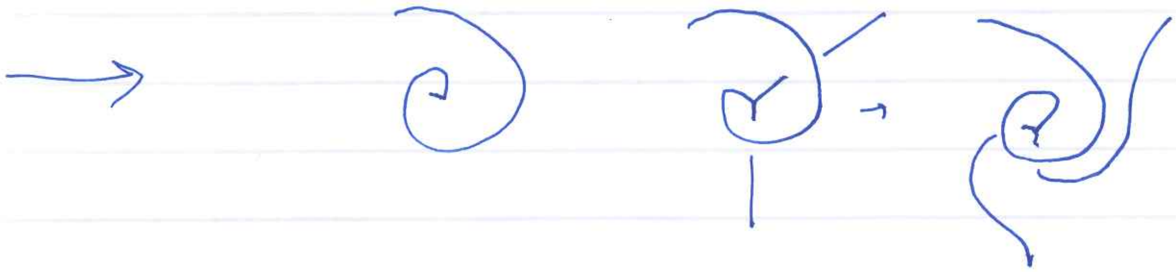


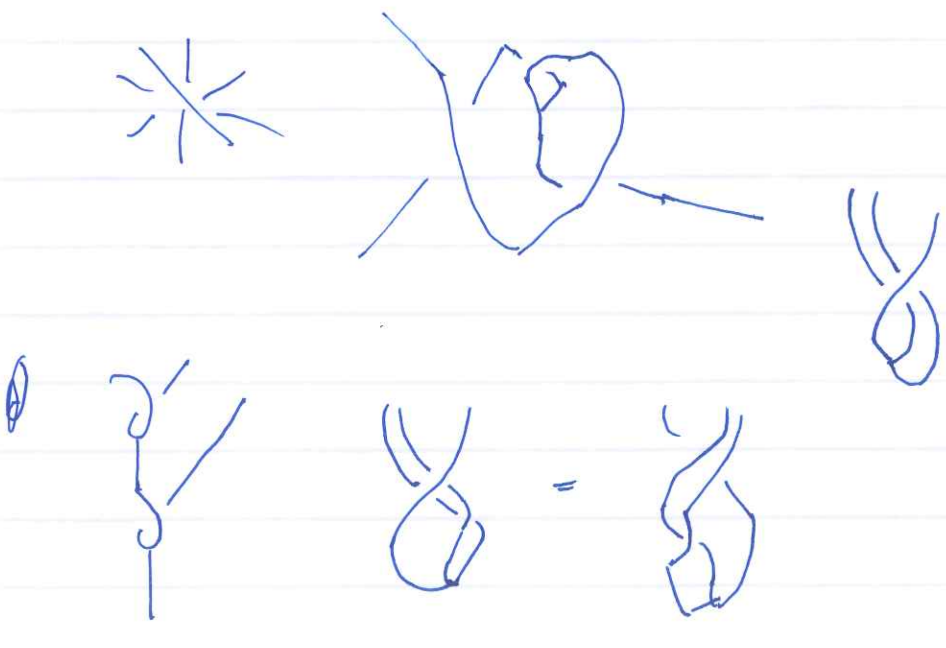
6
10
16

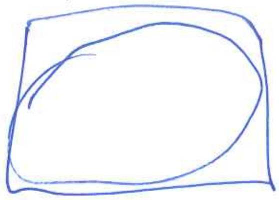
2hex + 4 pent. 4 strands











$$\left(\frac{\pi}{4}\right)^{n/2}$$

$$\varphi(x,y,z) = -\varphi(-x,-y,-z) = -\varphi(z,y,x)$$

$$\varphi(z,x,y) - \varphi(x,z,y) + \varphi(x,y,z) = 0$$

$$\varphi(x,y,z) - \varphi(w+x,y,z) + \varphi(w,x+y,z) - \varphi(w,x,y+z) + \varphi(w,x,y) = 0$$

$$\varphi(t,u,v) \overset{?}{=} \varphi(-t-u-v, v, u)$$

$$\varphi(a_1 t + a_2 u + a_3 v, \ldots, a_7 t \ldots + a_9 v)$$

I

a, b, c

$$g^2 a g^{-1} \cdot b g^{-7} = c g \cdot c$$

$$5^{12} = \frac{10^{12}}{2^{12}} = \frac{1}{4} 10^9$$

$$2^{10} \cdot \binom{21}{10} = 2^{30}$$

$$\alpha(x) \xrightarrow{d} \alpha(y) - \alpha(x+y) + \alpha(x)$$

$$F(x,y) \longmapsto F(y,z) - F(x+y,z) + F(x,y+z) - F(x,y)$$

Question: Suppose a functional identity in F holds for every $F = d\alpha$. Does it follow from $dF = 0$?

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longmapsto \begin{pmatrix} -x \\ x+y+z \\ -z \end{pmatrix}, \begin{pmatrix} z \\ -x-y-z \\ x \end{pmatrix}, \begin{pmatrix} x+y \\ z \\ -y-z \end{pmatrix}, \begin{pmatrix} -x-y-z \\ z \\ y \end{pmatrix}$$

$$\begin{pmatrix} -x \\ -y-z \\ z \end{pmatrix}$$

$$\begin{pmatrix} x+y \\ z \\ -y-z \end{pmatrix}$$

$$v_1 + v_2 = 3$$
$$v_2 + v_3 = 5$$
$$v_1 + v_3 = 4$$

1541

001 101 100 001

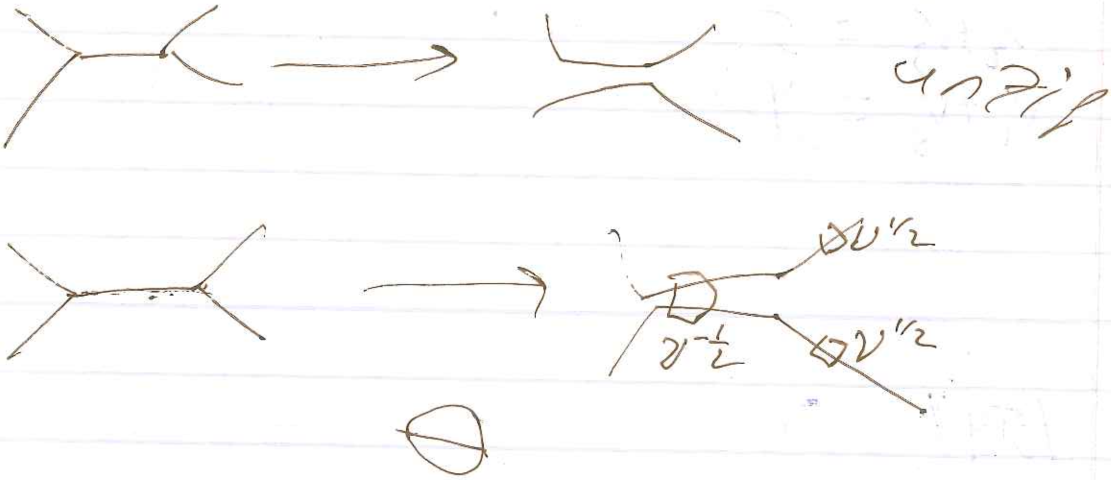
פרופ' ברור קריען שאמר

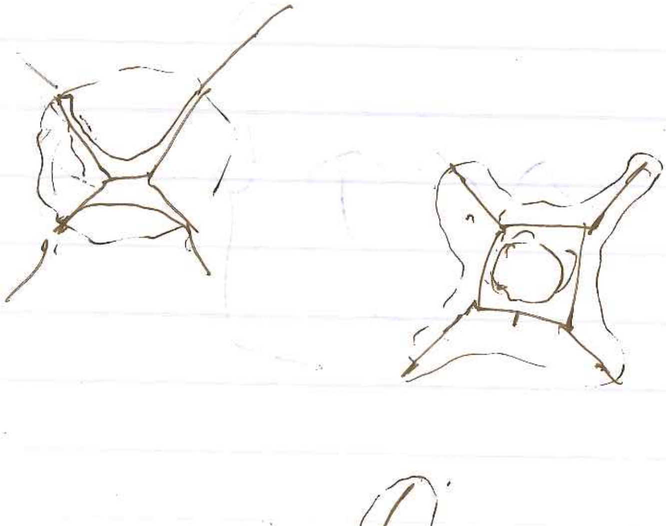
ש- $1+1 \neq 2$

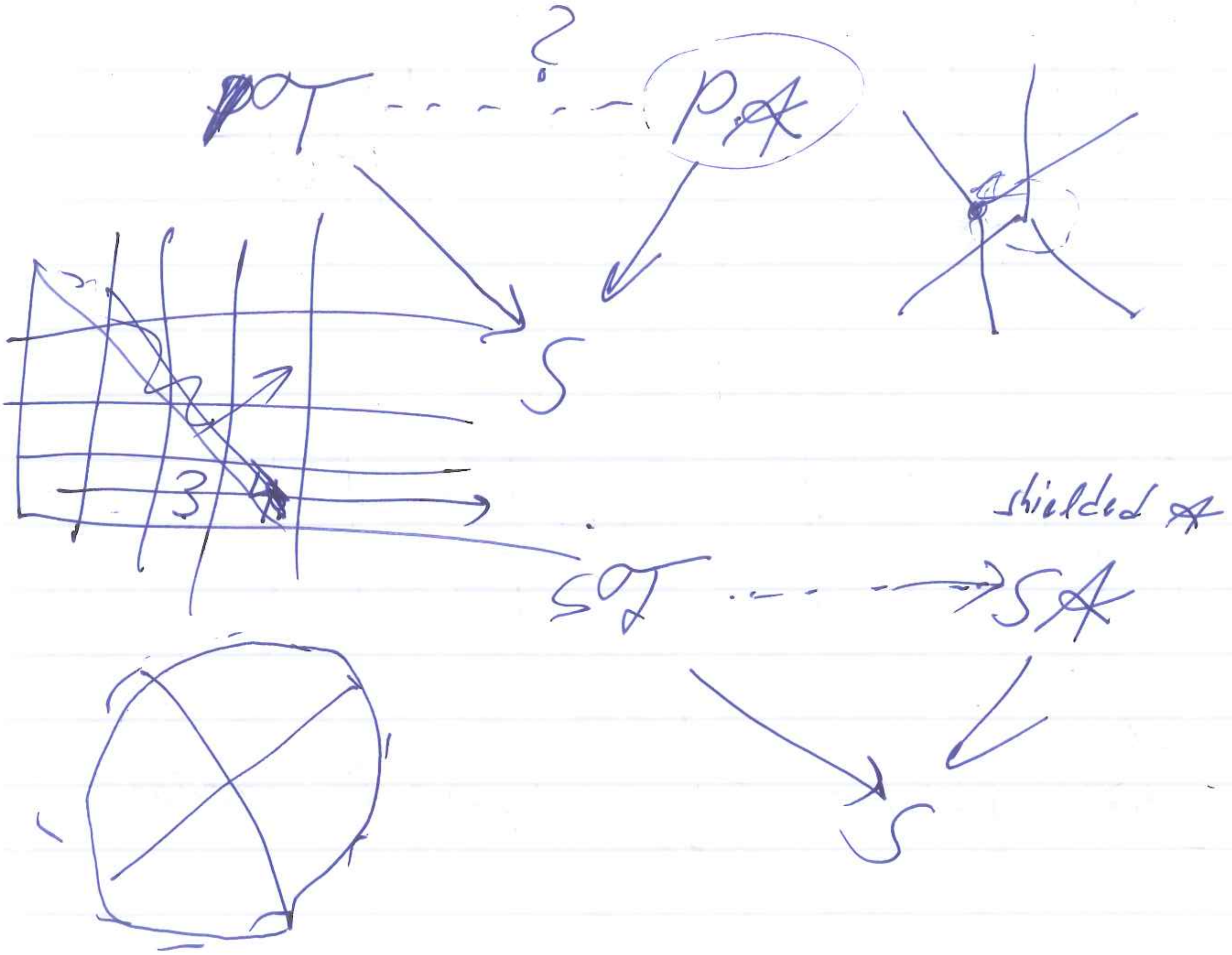
ברור

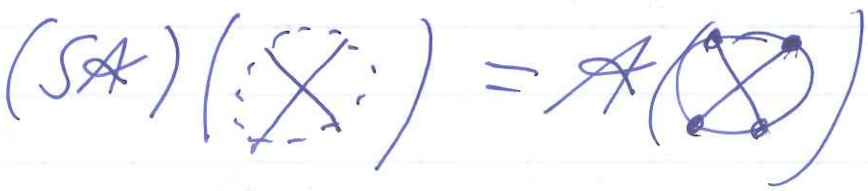
קיימת שאלת הבנה.

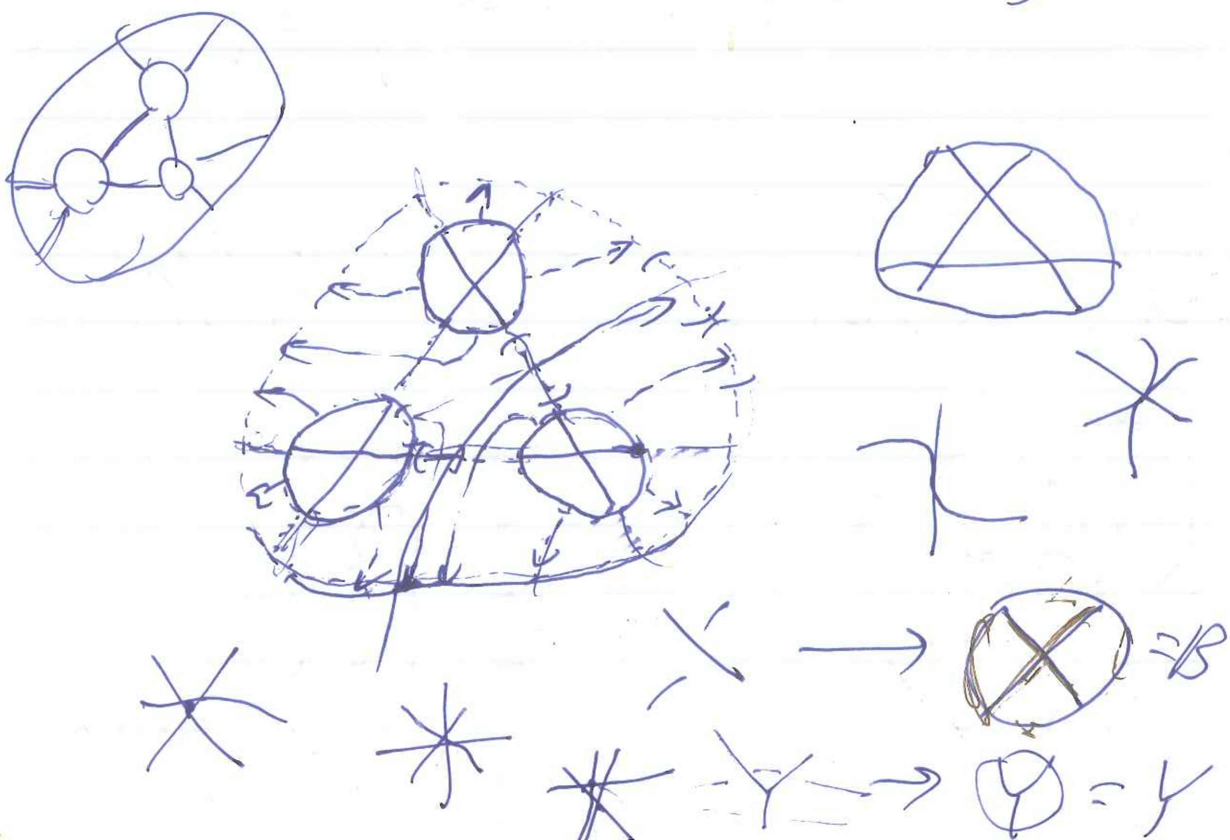
ברור.

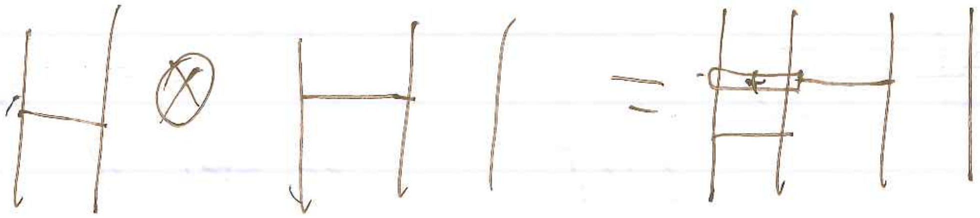

v^1/2
v^1/2
v^-1/2




POT
?
PA
S
shielded A
SOT
SA
S

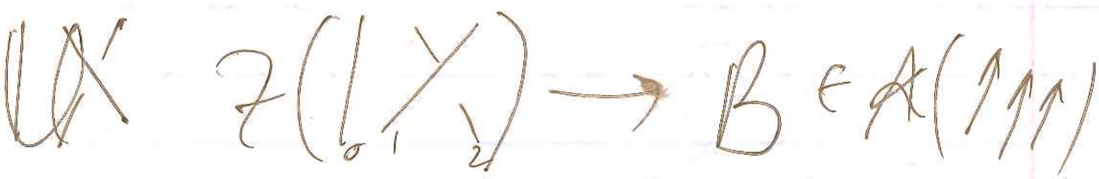

(SA)
= A

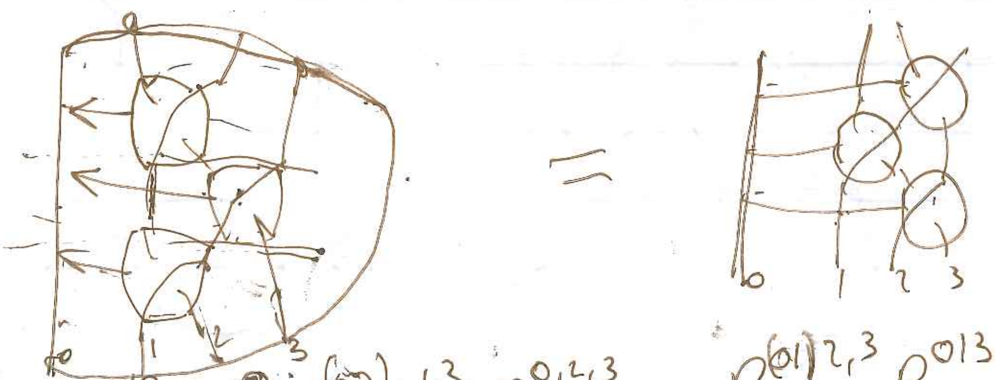




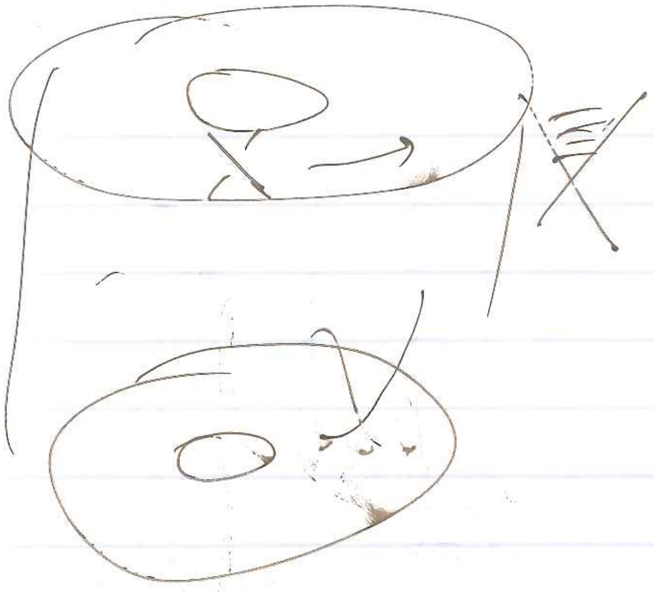
<u>Thm prop</u> ∃ Tensor Functor

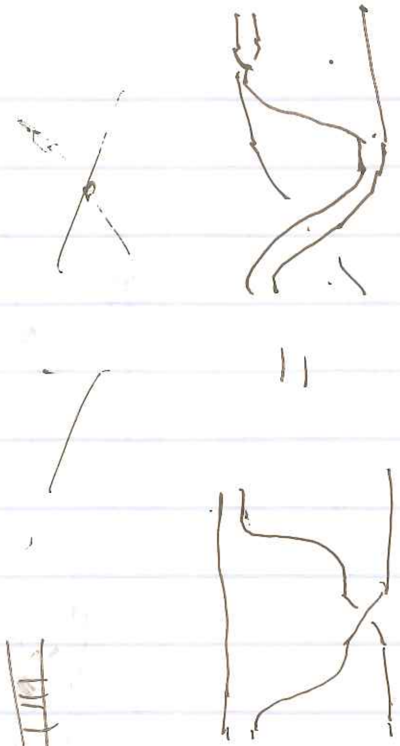
$z : AB \to AA$ s.t.

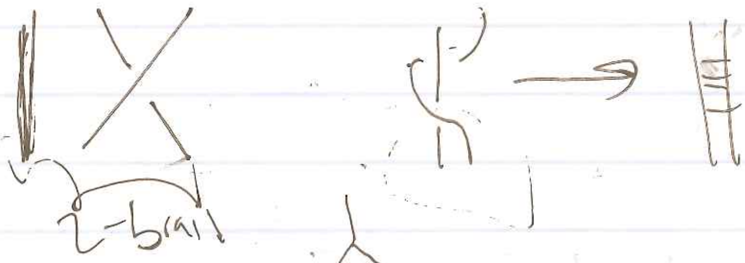


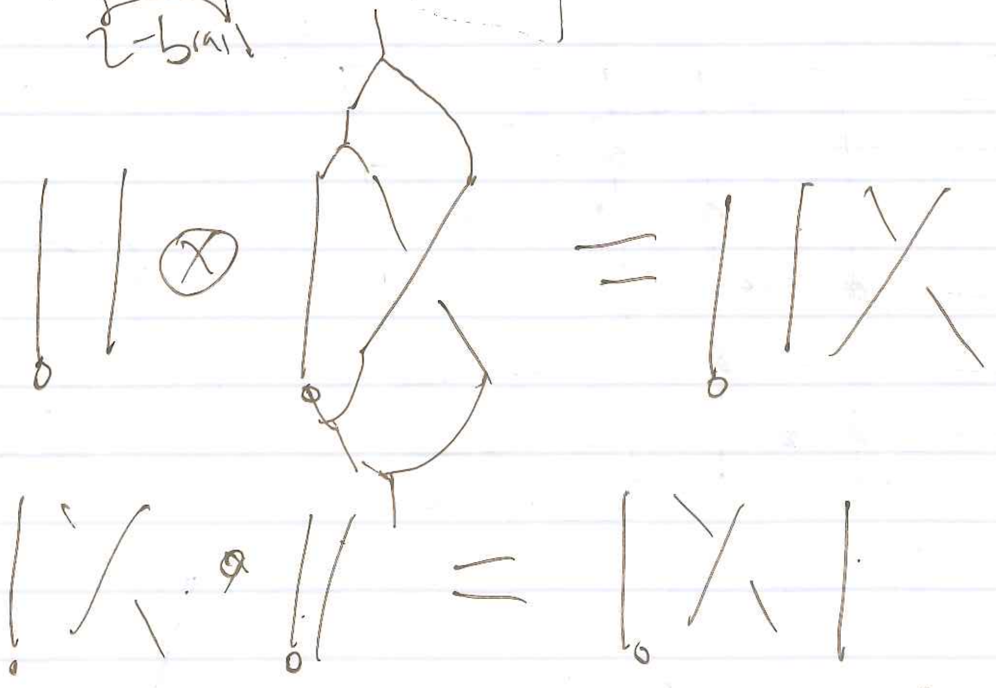


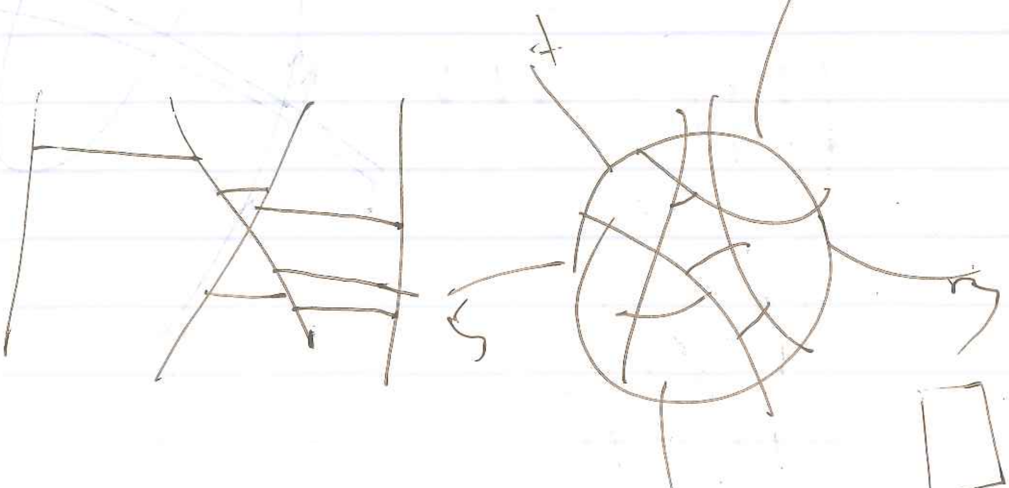
$B^{012} \cdot B^{01} B^{(02),1,3} \cdot B^{0,2,3} = B^{(01)2,3} B^{013} B^{(03)12}$

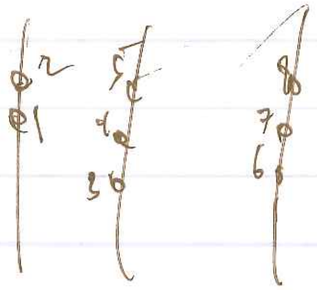




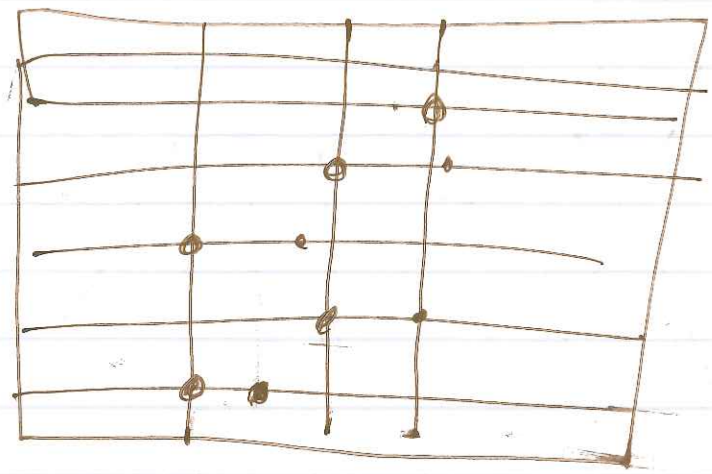

2-braid



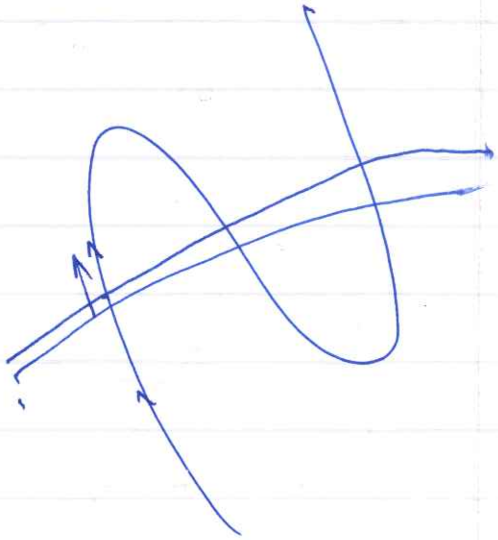




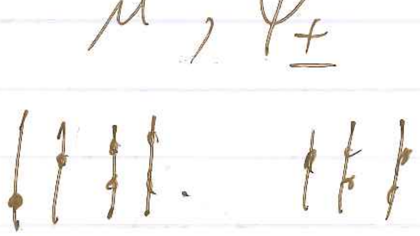
Fox–Neuwirth



Γ^7



$\mu, \Psi_\pm$



Ψ ρ

$$P \subset D$$

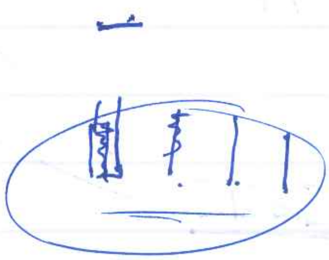
$$C = \frac{(D \setminus P) \times D \setminus P \setminus \Delta}{\mathbb{Z}/2}$$

$$S^1 \sim \frac{D \times D \setminus \Delta}{\mathbb{Z}/2}$$

$\mathbb{RP}^1$

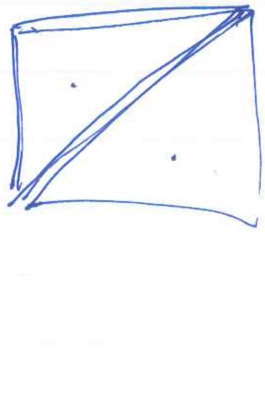
$+$

$$D \setminus P \times D \setminus P \to VS^1 \times VS^1 \to S^1 \times S^1 \xrightarrow{*} S^1 \; ?$$

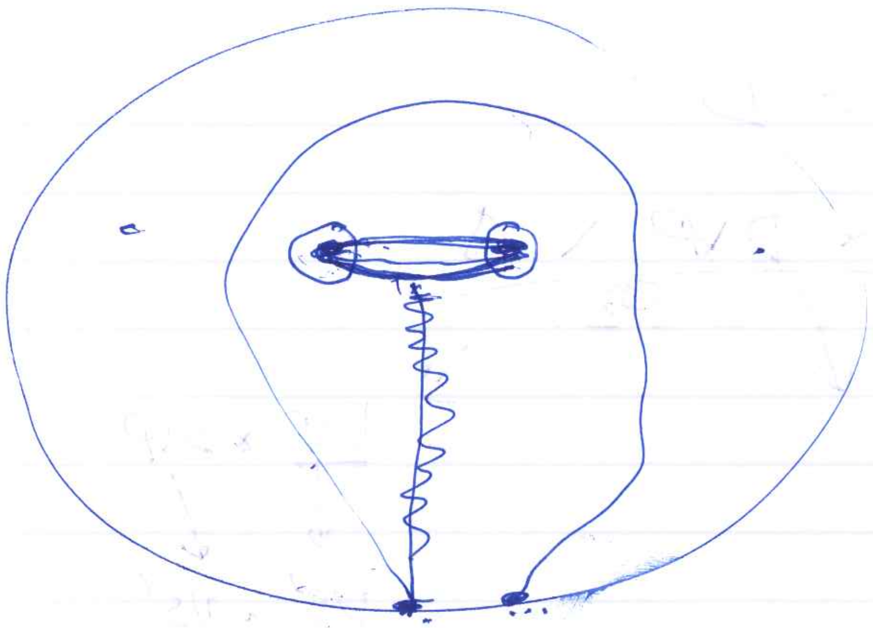


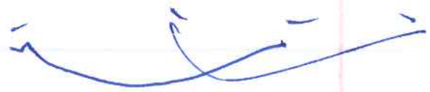
$$\begin{array}{ccc} * \in \tilde{C} & \longrightarrow & \mathbb{R}^2 \\ \downarrow & & \downarrow \\ C & \longrightarrow & S^1 \times S^1 \end{array}$$

$$\begin{array}{ccc} \tilde{C} & \dashrightarrow & \tilde{C} \\ \downarrow & & \downarrow \\ C & & C \\ & \searrow \; S^1 \times S^1 \; \swarrow & \end{array}$$



$H_2(\tilde{C})$





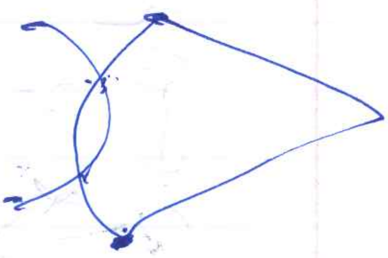
$\hat{C}$

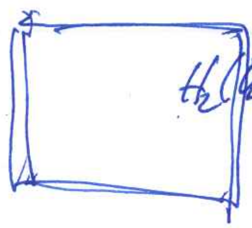
$\downarrow$

$\Sigma_N \quad \frac{I \times I \setminus \Delta}{\mathbb{Z}/2} \simeq D^2 \longrightarrow C \supset U$

$\Sigma_F \quad I \times I \nearrow$

$\Sigma_N \cap \Sigma_F$





$H_2(\hat{U}) \to H_2(\hat{C}) \to H_2(\hat{C}, \hat{U}) \to H_1(\hat{U})$

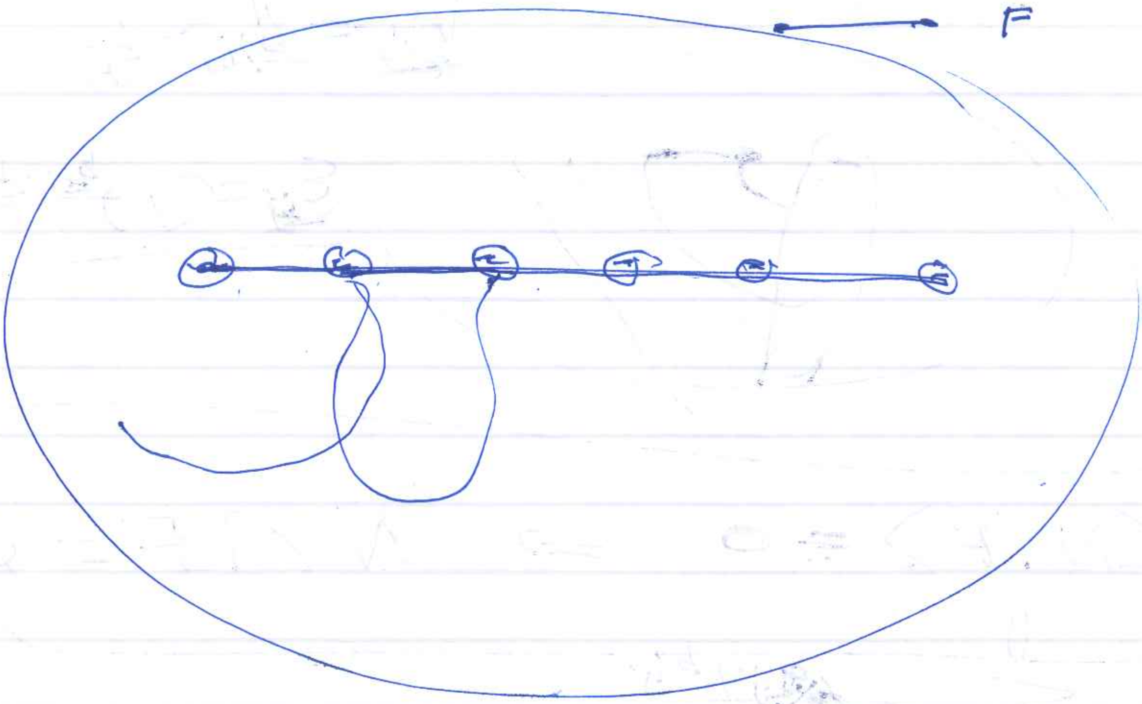
F

σ is in kernel of reps

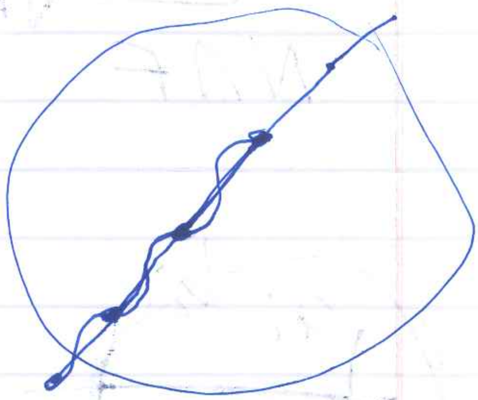
$\sigma[\Sigma_F] = \sigma[\Sigma_F]$

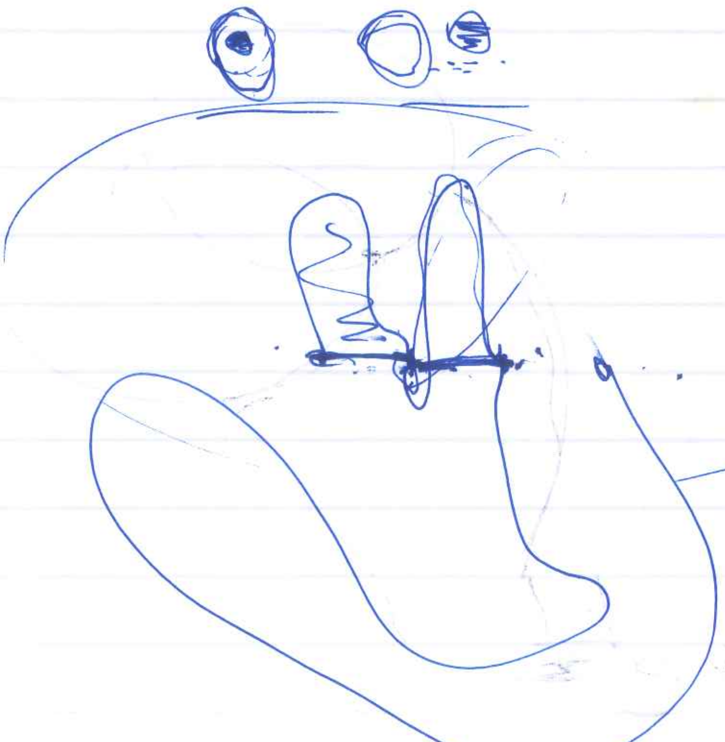
$\cup\!\!\!\!\!\!\parallel$

$\sigma(F) = F$



F





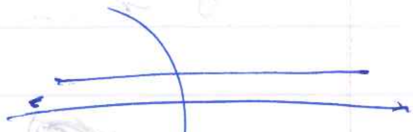
$\pi_1(D \setminus \cdots\cdots)$

$\subset$

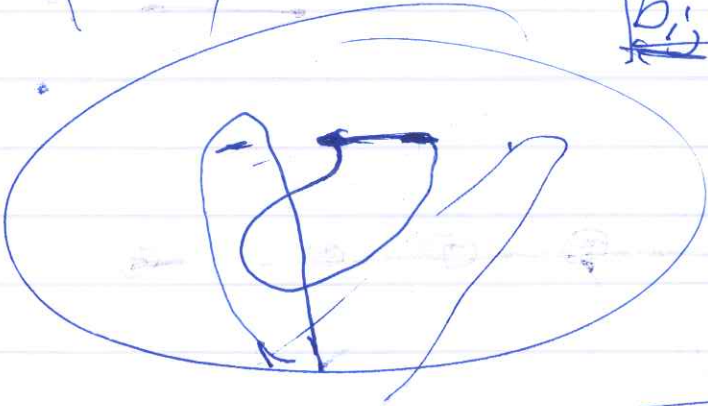
$\pi_1(D \setminus \cdots\cdots)$

$a_{ij} = a_{ii} = a_{jj}$

$a_{ij} = a_i + a_j + a$



$b_{ij} = b_{ii} = b_{jj}$



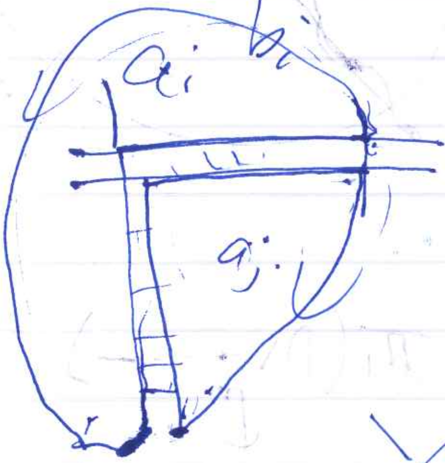
$\varepsilon_{ij} = (-1)^{b_{ii} + b_{ij} + b_{jj}}$

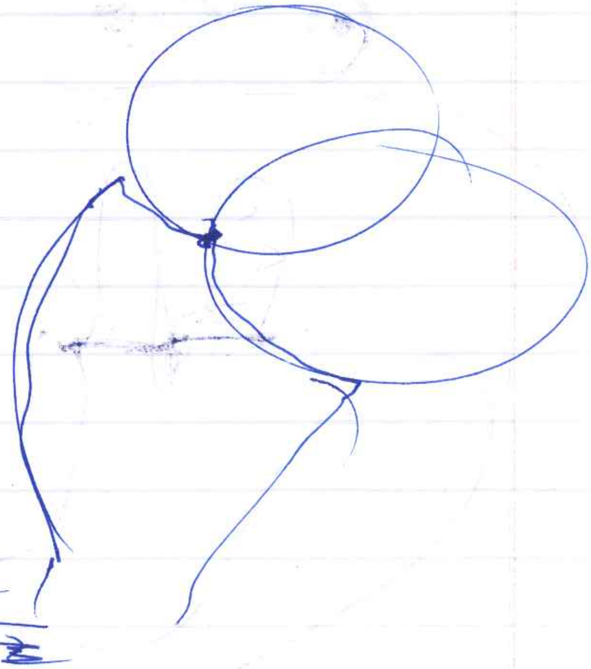
$\langle N . F \rangle = 0 \Rightarrow N \cap F = \emptyset$

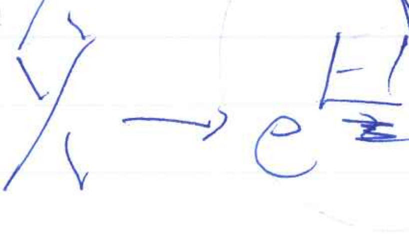
$\|$

$\sum \varepsilon_{ij} \phi^{b_{ij}}$

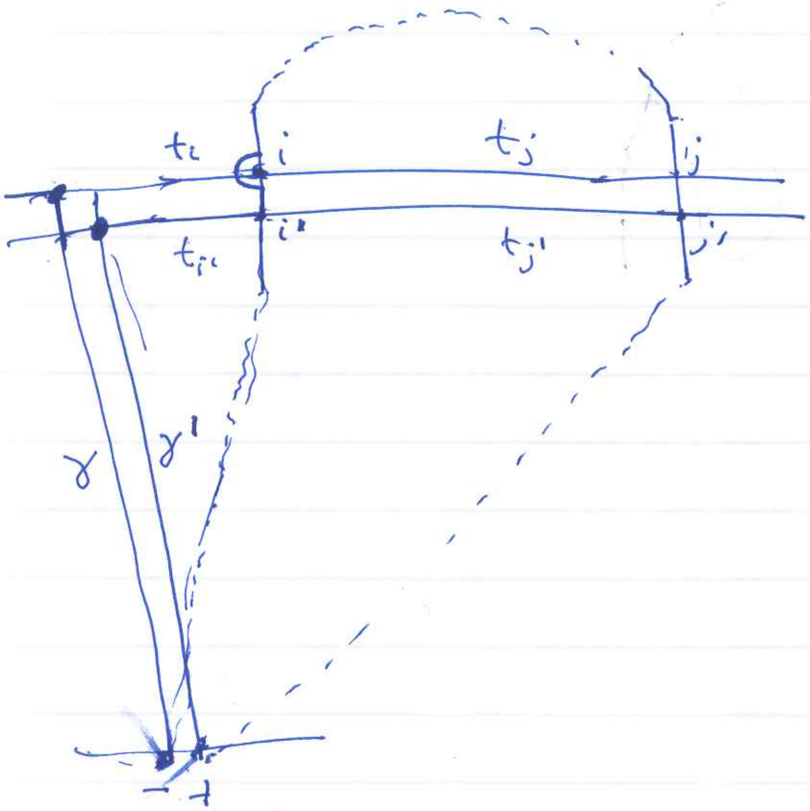
$N \cap F$

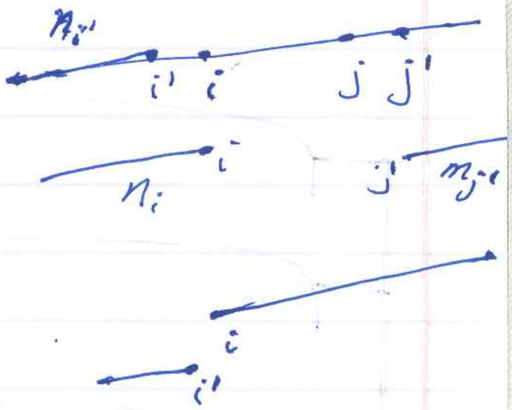






$b_{ij} - b_{ii} > 0$

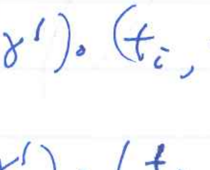


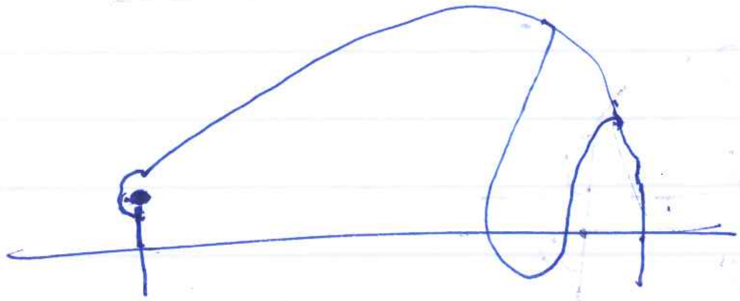


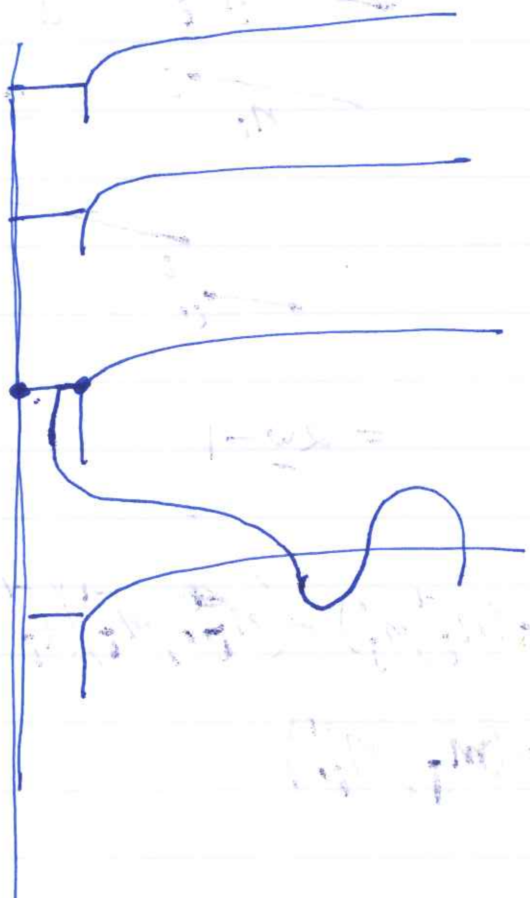
$= 2w - 1$

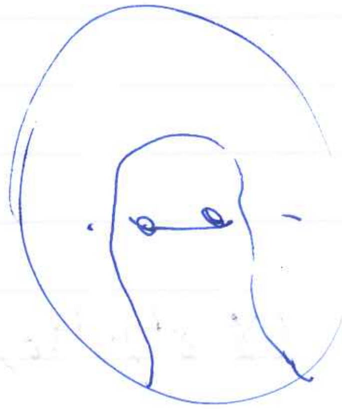
$(\gamma\ \gamma')\circ(t_i, t_{j'})\circ(n_i^{-1}, m_j')\circ(n_{i'}, m_i^{-1})(t_i^{-1}\ t_{i'}^{-1})(\bar{\gamma}$

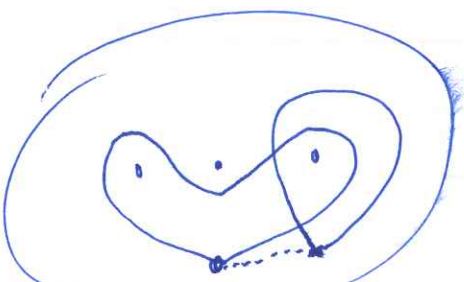
$(\gamma\ \gamma')\circ(t_i, t_{i'})\circ(m_j, n_{i'}^{-1})$

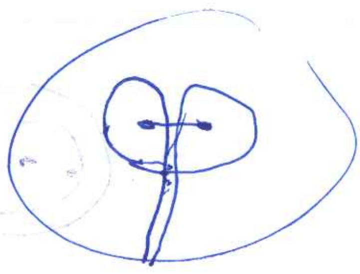


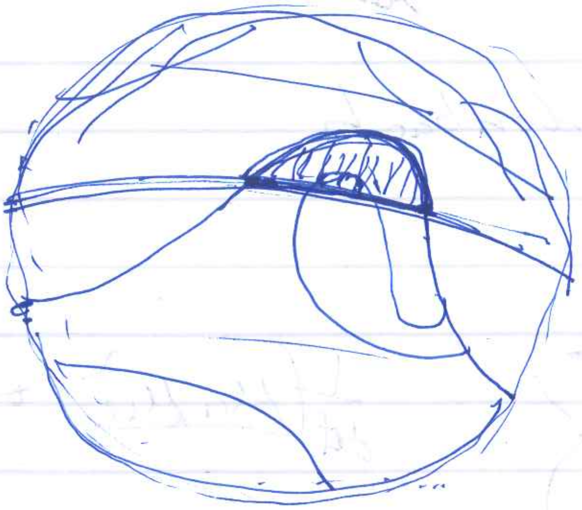


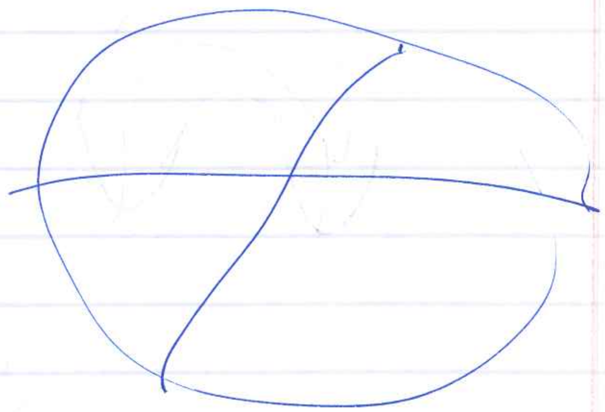




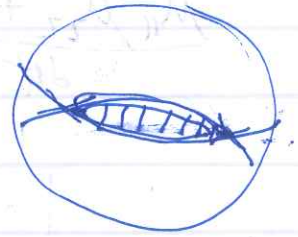


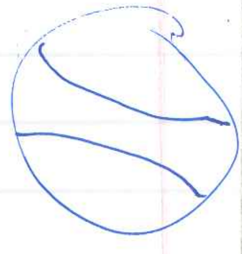






Elitzur





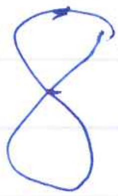
Kutasov

Berkooz

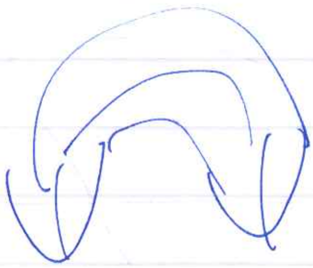
Elitzur-Moore-Schwimmer-
Seiberg

$G_{\mu\nu} = T_{\mu\nu}$

— Laudenbach



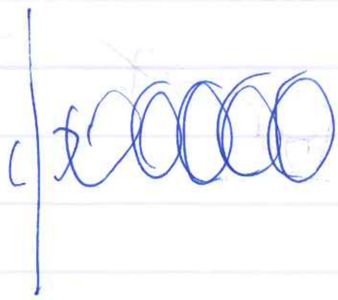
$$\frac{1}{\det}\left(\mu_{11}\mu_{12}^2 + \mu_{22}\mu_{12}^2\right)$$

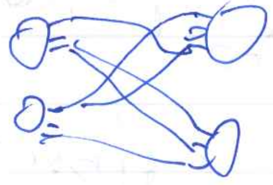


$$\begin{pmatrix}\mu_{11} & \mu_{12}\\ \mu_{12} & \mu_{22}\end{pmatrix}^{-1} =$$

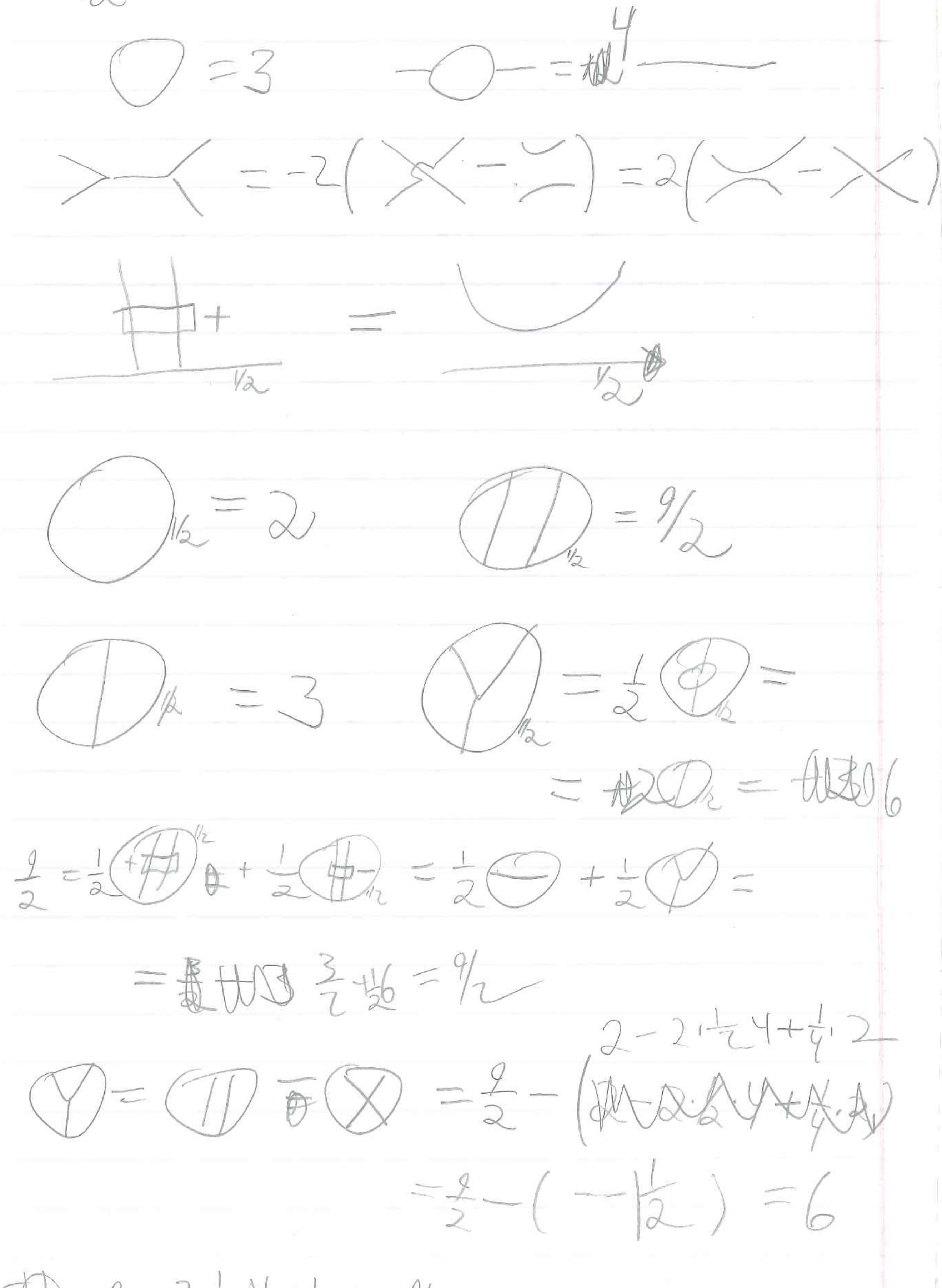
$$= \begin{pmatrix}\mu_{22} & -\mu_{12}\\ -\mu_{12} & \mu_{11}\end{pmatrix}$$

$$\frac{\mu_{11}^2\mu_{22} + \mu_{11}\mu_{22}^2}{\det} = \frac{\mu_{11}\mu_{22}(\mu_{11}+\mu_{22})}{\det}$$





sl_2:



= 3
= 4
= −2(
) = 2(
+
1/2
= 2
= 9/2
= 3
= 1/2
= 6
9/2 = 1/2
+ 1/2
= 1/2
= 3/2 + 6 = 9/2
2 − 2 · 1/2 · 4 + 1/4 · 2
= 9/2 − (
= 9/2 − (− 1/2) = 6
= 8 − 2 · 1/2 · 4 + 1/4 · 2 = 9/2

$$ax^2+bx+c=0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$1 \longrightarrow t_1$$

$$t_1 \longrightarrow -t_1 + c_1/2$$

$$\begin{pmatrix} 0 & c_1/2 \\ 1 & -1 \end{pmatrix} \qquad P = c_1/2 \qquad S = -1$$

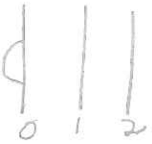
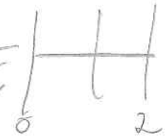
$$\lambda^2 + \lambda - \frac{c_1}{2} = 0$$

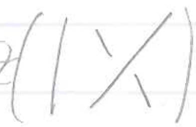
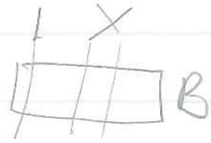
$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1+2c_1}}{2}$$

$$(\sqrt{c} \pm 1)^2 = c + 1 \pm 2\sqrt{c}$$

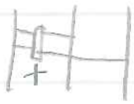
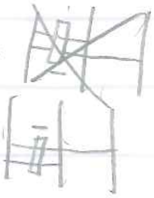
$$S = 2c$$

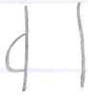
$$P = c^2$$

$c_2 =$  $id_2 =$  $t_2 =$ 

$\tilde{B} = \Sigma$  $=$ 

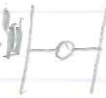
$B = b_0(c_2) \cdot + b_1(c_2) \cdot$  $+ b_2(c_2) \cdot$  $+$

$+ b_3(c_2) \cdot$  $+ b_4(c_2) \cdot$ 

$\Delta(c_0) =$  $+$  $+ 2$ 

$= c_1 + \frac{3}{2} id_1 + 2$ 

$t_1^2 =$  $= \frac{1}{2}$  $+ \frac{1}{2}$  $=$

$= \frac{1}{2} c_1 + \frac{1}{2}$  $= \frac{1}{2} c_1 - \frac{1}{4}$  $=$

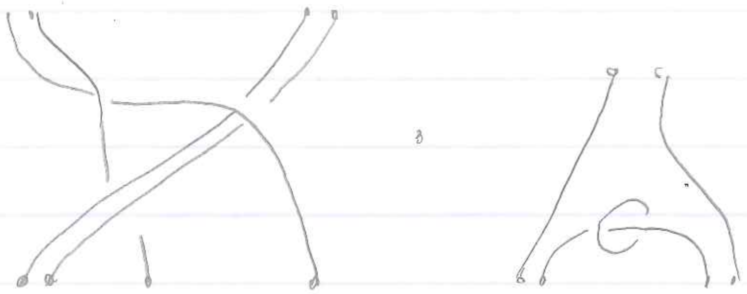
$= \frac{1}{2} c_1 -$  $= \frac{1}{2} c_1 - t_1$

$t_1^3 =$  $= t_1\left(\frac{1}{2} c_1 - t_1\right) = \frac{1}{2} c_1 t_1 - \left(\frac{1}{2} c_1 - t_1\right)$

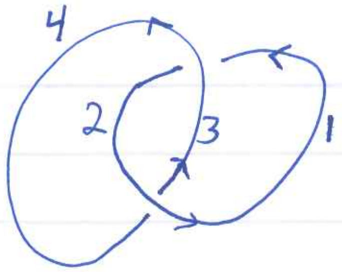
$= \frac{1}{2} c_1 (t_1 - 1) + t_1 = t_1\left(1 + \frac{1}{2} c_1\right) - \frac{1}{2} c_1$

$t_1^4 = t_1(-1 - c_1) + \frac{1}{2} c_1 \left(1 + \frac{c_1}{2}\right)$

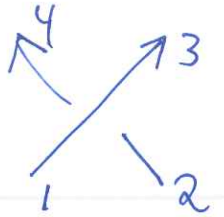
$t_1^5 = t_1\left(\frac{c_1^2}{4} + \frac{3}{2} c_1 + 1\right) - \frac{c_1}{2}(1 + c_1)$



=



→



→ X[1, 2, 3, 4]

link = Link[S[1,2], S[3,4], X[3,1,4,2], X[2,4,1,3]]

CCube[link] ⟼ CCube[

{{A B A A ...} →

{V[A, B, B, A ...] ⟶ Cycles[1, 3, ...]

⋮

},

{E[i, A, B, ...] → Merge[f1, f2, t]
Split[F, t1, t2]

⋮

}]

Khovanov's Categorification, July 30 2001.

$R = \mathbb{Z}[c]$ $\deg c = 2$ (may set $c=0$)

$A = R\langle e, X\rangle$ $\deg \begin{matrix} e = 1 \\ X = -1 \end{matrix}$ commutative, cocommutative

with $eX = X$ $e^2 = e$ $X^2 = 0$

$\Delta(e) = e\otimes X + X\otimes e + cX\otimes X$ $\Delta(X) = X\otimes X$

$\epsilon(e) = -c$ $\epsilon(X) = 1$

$L =$ 

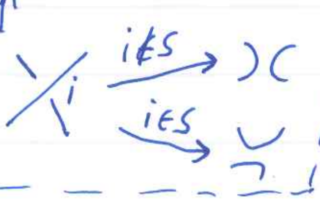
$\mathcal{X} = \{i : i \text{ is a xing}\}$

$x(L) :=$ # of under xings

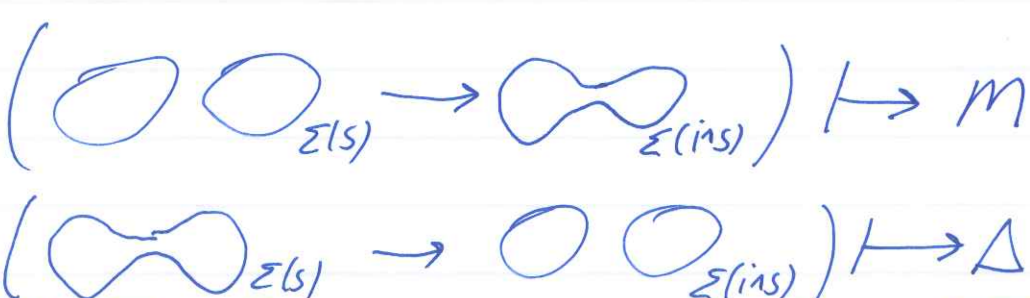
$y(L) :=$ # of over xings

$\Lambda(\mathcal{X}) :=$ "the anti-symmetric power set of $\mathcal{X}$"

$\Lambda(\mathcal{X}) \ni s \longmapsto \Sigma(s)$ a union of cycles $\longmapsto A^s := A^{\otimes|\Sigma(s)|}$

via 

$\ell = (s,i)$, $s \in \Lambda(\mathcal{X})$, $i \notin s$ $\longmapsto T_\ell : A^s \to A^{i \wedge s}$ by



$\overline{C}^r(L) := \bigoplus_{\substack{s\in\Lambda(\mathcal{X}) \\ |s|=r}} A^s$; $d : \overline{C}^r \to \overline{C}^{r+1}$ by summing T_ℓ over all relevant edges ℓ

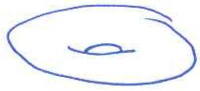
$C(L) := \overline{C}(L)[x(L)]\{2x(L) - y(L)\}$ where $[\_]$: shift down r-deg, $\{\_\}$: shift down deg

$H(L) = H(C(L)) = \bigoplus H^{r,j}(L)$

Thm $K(L) = (1-q^2)\sum_{r,j} (-1)^r q^j \dim_q H^{r,j}(L)$

where $K(L) = (-1)^{x(L)} q^{y(L)-2x(L)} \langle L\rangle$

$\langle \bigcirc \rangle = q + q^{-1}$; $\langle \times \rangle = \langle \asymp \rangle - q\langle)(\rangle$



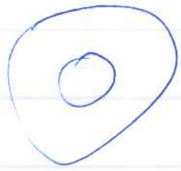
$$(z,1) \longmapsto (0,1)$$

$$(1,z) \longmapsto (1,0)$$

$$\begin{pmatrix}1 & z\\ z & 1\end{pmatrix}^{-1} = \frac{1}{1-z^2}\cdot\begin{pmatrix}1 & -z\\ -z & 1\end{pmatrix}$$

x

z/x



$$\frac{1}{1-z^2}\begin{pmatrix}x - z^2/x\\ -zx + z/x\end{pmatrix}$$

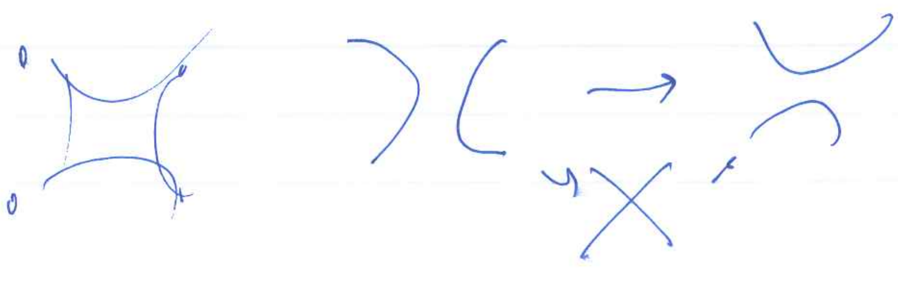
$x = z, .. 1$

$$\begin{pmatrix}z^2 & 1 & \cdot\\ 1 & z^2 & \cdot\\ \cdot & \cdot & 1\end{pmatrix}^{-1}\begin{matrix}z e^{t\log z}\\ z e^{-t\log z}\\ z^2\end{matrix}$$

$z = 0..1/e$

$t = -1, 1$

$$\frac{z}{1-z^4}\begin{pmatrix}z^2 & -1\\ -1 & z^2\end{pmatrix}\begin{pmatrix}e^{t\log z}\\ e^{-t\log z}\end{pmatrix}$$

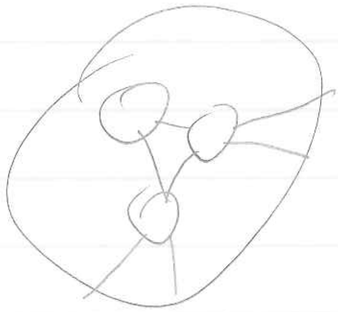


Algebraic Structures on Knotted Objects, Davis Aug 13, 200

0. Intro on life etc.

1. Introon B_y

2. The PAJ story

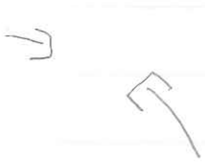


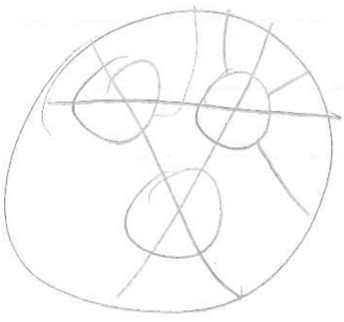
To do by Monday:

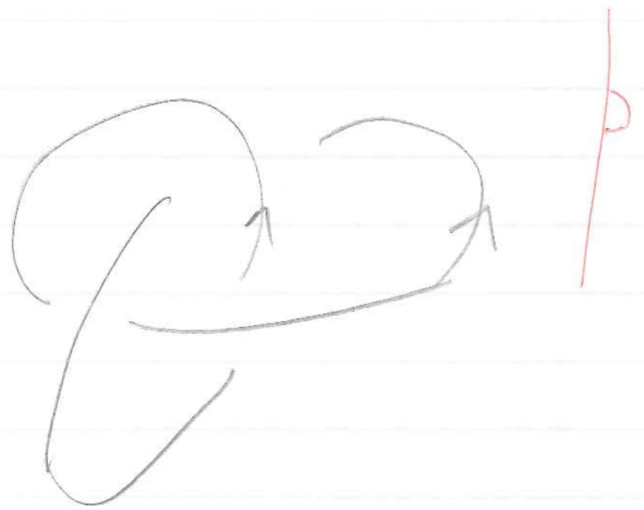
1. a syzygy for KTG

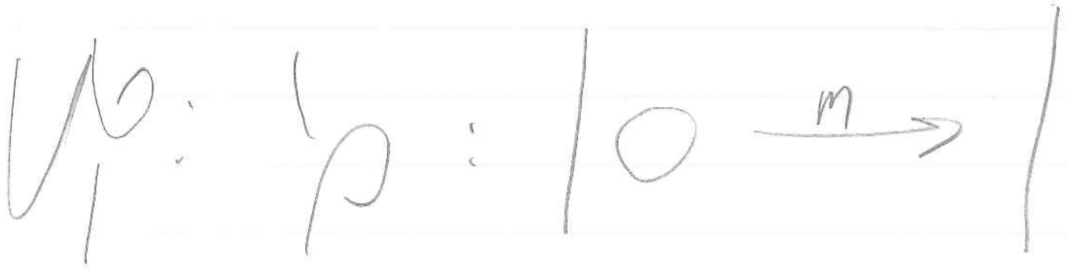
2. Composition of shielded tangles

3. list of live ends.









$X \quad v_+ \longrightarrow X$

$v_- \otimes v_- \longrightarrow 0$

$v_+ \otimes v_- - v_- \otimes v_+ \longrightarrow 0$

$v_- v_- v_+$

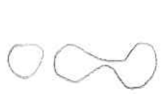
$v_+ v_+ = \alpha v_+ \quad v_+ v_- = \beta v_- \quad v_- \otimes v_- = 0$

$\Delta v_+ = \gamma (v_+ \otimes v_- + v_- \otimes v_+)$

$\Delta v_- = \delta v_- \otimes v_-$

associativity/commutativity $\Rightarrow \alpha\beta = \beta^2 \quad \beta = 0$ or $\alpha = \beta$

Coassociativity/cocomm $\quad \gamma^2 = \gamma \Rightarrow \gamma = 0$ or $\gamma = \delta$

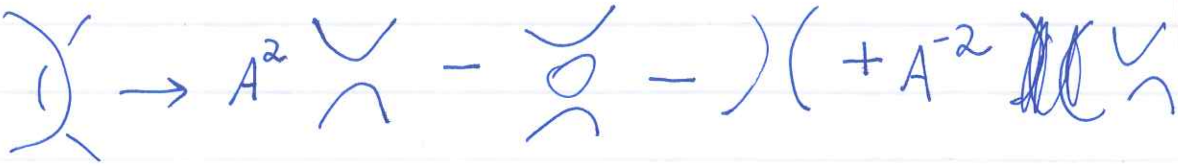


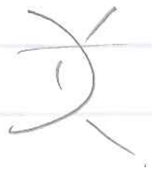
$\Delta \circ m = (m \otimes 1)(1 \otimes \Delta)$

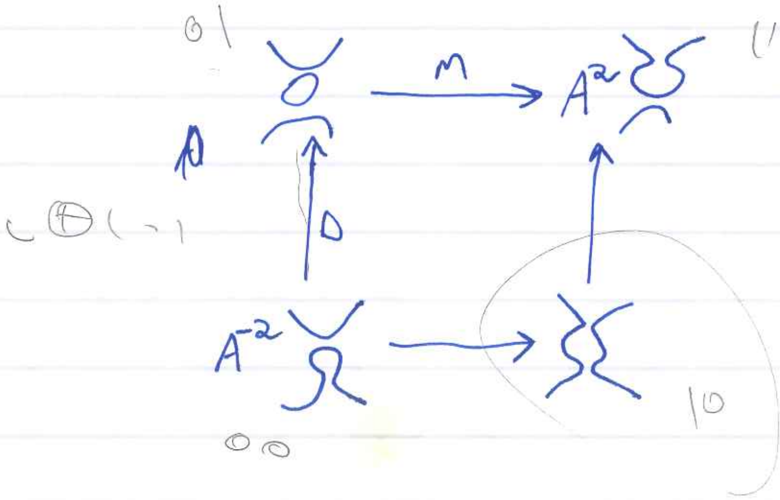
$v_+ v_+ \rightarrow \alpha\gamma (v_+ \otimes v_- + v_- \otimes v_+)$;

$v_+ v_- \rightarrow \alpha\delta\, v_- \otimes v_-$;

wrong handedness?



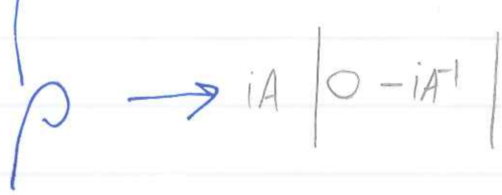


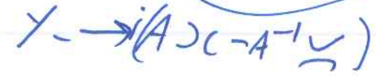


$\Delta(u) = u \otimes d + d \otimes u$ $\qquad u \cdot u = u$

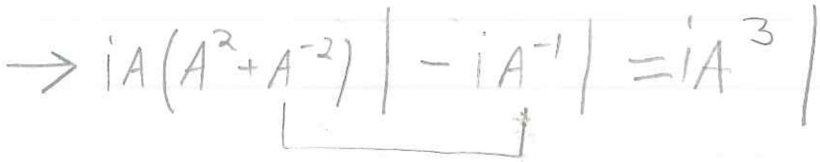
$\Delta(d) = d \otimes d$ $\qquad u \cdot d = d$

$\qquad d \cdot d = 0$





$O \rightarrow A^2 + A^{-2}$



Categorification For/by the Feeble minded*

Dror Bar-Natan, Calgary Aug 24 2001

http:// - - - -

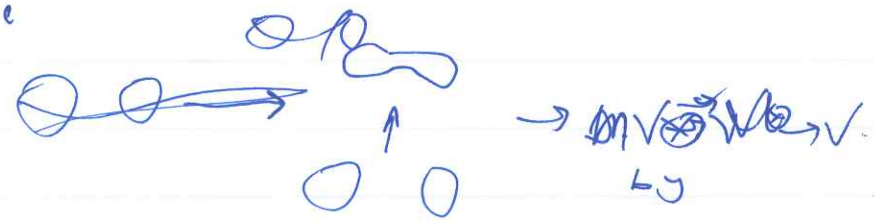
The Kauffman bracket:

In detail:

~~Khovanov's categorification~~

Khovanov's categorification:

where



nontrivial, lucky, non-local, unstable and orthogonal to all we know.

Bravehearts & bigheads: Khovanov's arXiv:math.QA/9908171 & 0103190

Algebraic structures on knotted objects, Calgary Aug 23, 2001

j/w/ Dror Bar-Natan

~~Goal: Find a "good" presentation of knot theory in terms of generators and relations~~

~~good: *~~

1. Goal: Find some "good" algebraic structure on some set of knotted objects, with respect to which knot theory becomes finitely presented.

2. Why? — to get invariants

3. What's "good"?
   - * has parallel in chord diagrams
   - * Allows for the "why" to work
   - (there's much more to say: syzygies, obstructions/homology, symmetrization/formal PBW, Anonymization, jelly beans)

There are several approachs; lets concentrate on just one.....

} on board, 5 min

4 KTG — 3 min

5. Moves — 5 min

6. Domino theory on  — 10 min

7. The dual pachner move (full drawing
   - * Same as Associators & Drinfeld's pentagon
   - * related to 6j symbols & the Biedenharn-Elliot identity
   - * dual of Pachner moves

} on board, 7 min

G_* s. group

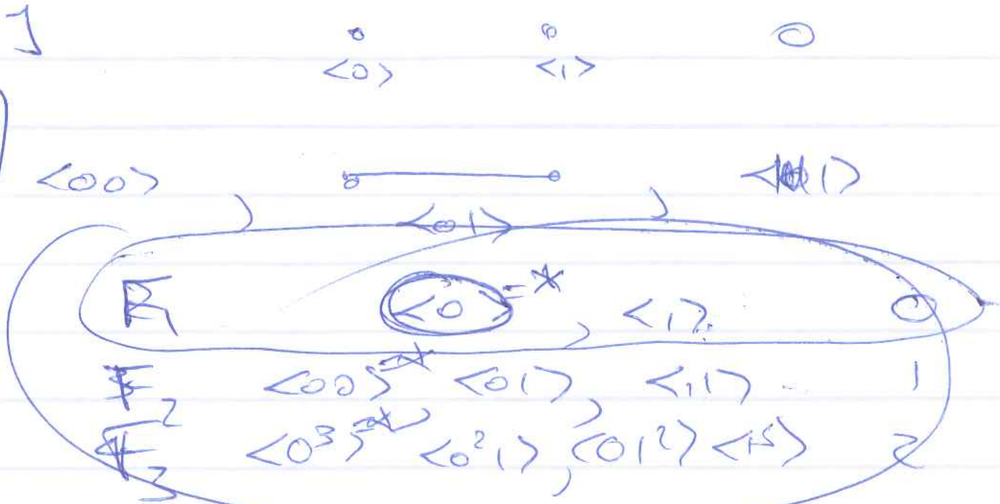
$G_0 = \{\text{point}\}$ / reduced

$\underline{\Omega(G_*)}_N = \ker(d_0 : G_{N+1} \to G_N)$

$1 \to \ker(d_0) \to \underline{G_{N+1}} \xrightarrow{d_0} \underline{G_N} \to 1$

$1 \to F_{N+1} \to \pi_1 F(\mathbb{R}^2, N+2) \xrightarrow{\text{proj}} \pi_1 F(\mathbb{R}^2, N+1) \to 1$

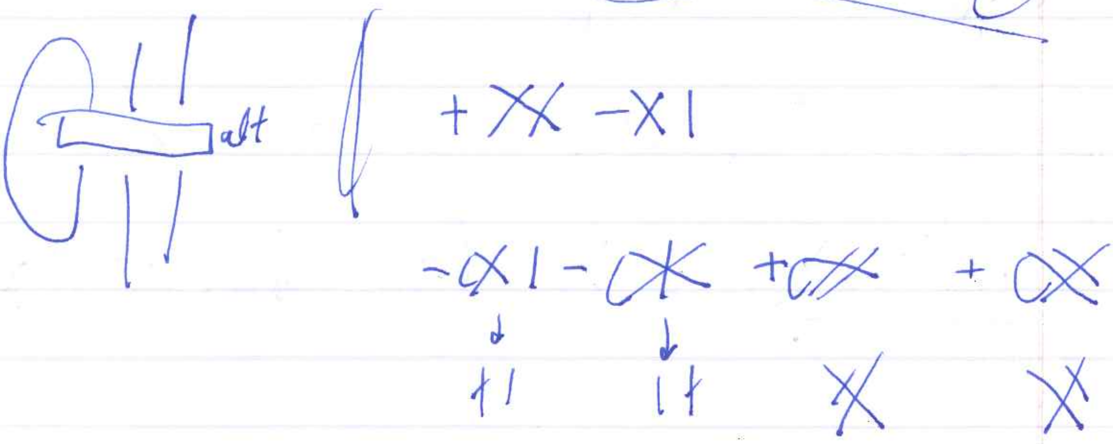
$\Delta[1]$



$\|\Omega(G_*)\|_{\text{sup}} \simeq \Omega_{\text{top}} \|G_*\|$

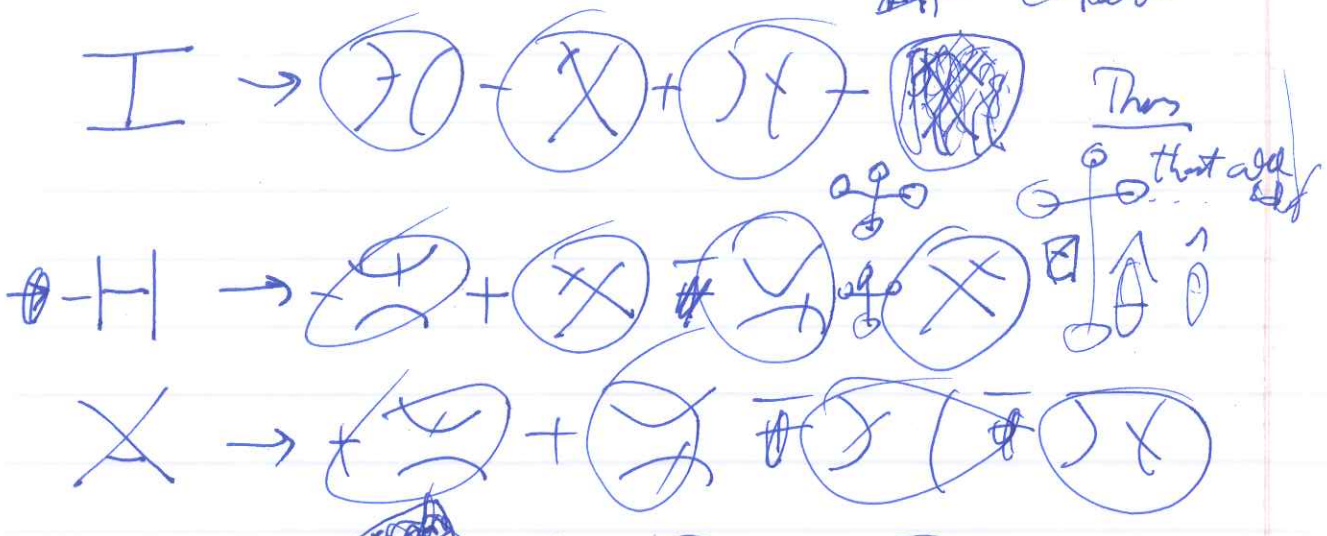
$F(\Delta[1]) \xrightarrow{\cong} \Omega(G_*)$

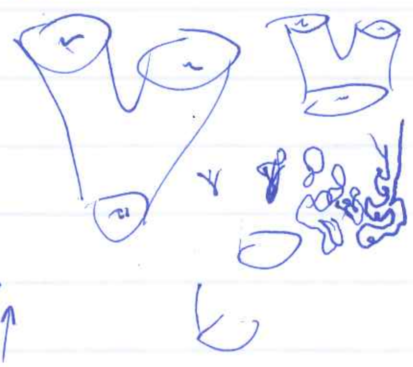
Ω (Artin's pure br. group)

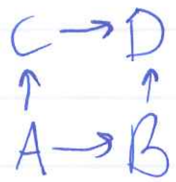

alt

$I = H - X$

$I - H + X = 0$

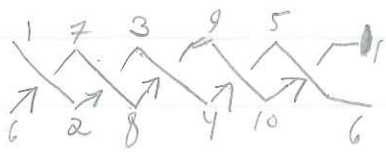




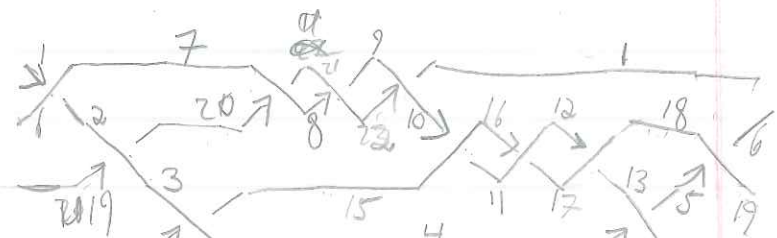


Li

$5_1 : 1^5$



$10_{132} : 1^{-1} 2 3 1^3 2^{-3} 3 2$



Knotted trivalent graphs, Tetrahedra & Associators.

Associators, parenthesized tangles —Ohtsuki / J. Murakami→ invariant of knotted trivalent ribbon graphs [Sorry, Christine.]

on board

D. Thurston, DBN: 1. This is more fundamental

2 ⇐ exists

3. More 3D; more symmetrical.

4. xing-centric as opposed to Ass-centric

5. related to Turaev-Viro & to 6j symbols.

6. provides us with much HW.

<u>Thm</u> (J. Murakami, Ohtsuki) (using Ass, PaT) There exist a universal system of invariants $Z_\Gamma : K(\Gamma) \longrightarrow \mathcal{A}(\Gamma)$ well behaved under

$\boxed{\nu = 1}$

0. Relabeling
1. dropping an edge
2. unzipping an edge
3. external connected sum

1. explanation of all notions in the theorem.
2. $\{Z_\Gamma\}$ is determined by

$$\text{Tet} = Z_\Delta(\Delta) \in \mathcal{A}(\Delta)$$

and

$$\text{Mob} = Z_{\circ}(\circ) \in \mathcal{A}(\circ)$$

3. The dual Pachner move
4. equivalence with pent.
5.
6. relation with T.V.

Knot invariants, Associators and a strange breed of Planar Algs.

Kyoto Sep. 19, 2001

5. 1. Introduction: Alg. str, etc.

5. 2. Planar Algebras.

3. 3. The No-go.

12. 4. Def'n of the shielded PA.

5 5. Existance thm & sketch of proof.

5 6. Relations & Syzygies

7. Why is it better?

a. elegantler and naturaler.

5 b. Allows more reductions.

8. The braid version:

5 a. The tensor cat. of braids and the no go.

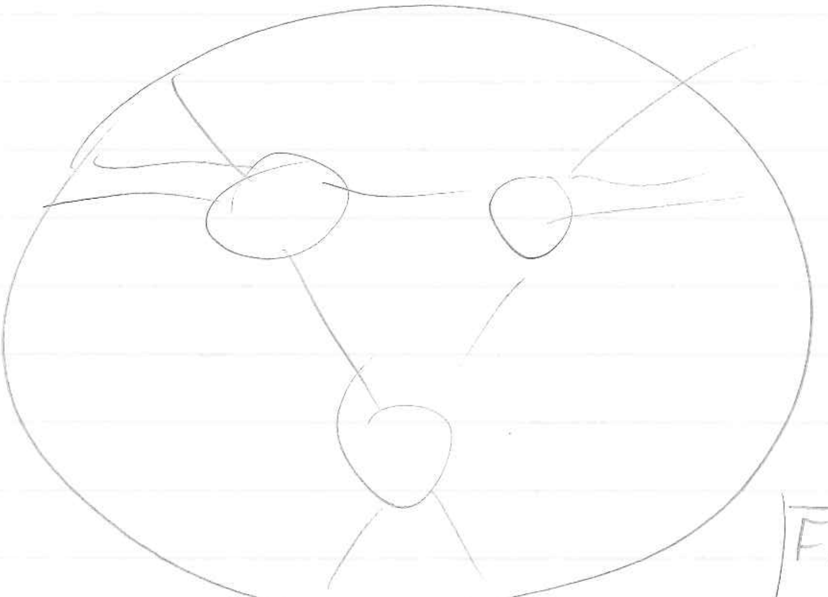
10 b. Annular braids & the "go".

5 c. Flash the open problems about possibility of solution.

On board:

Knot invariants and a strange breed of planar Algebras
by DBN, Dylan Thurston and the ~~NFAA~~ NAAF

Key point: There exists a skeleton-preserving planar-Algebra map $\mathcal{T}_0 \longrightarrow SA$



As in slide.

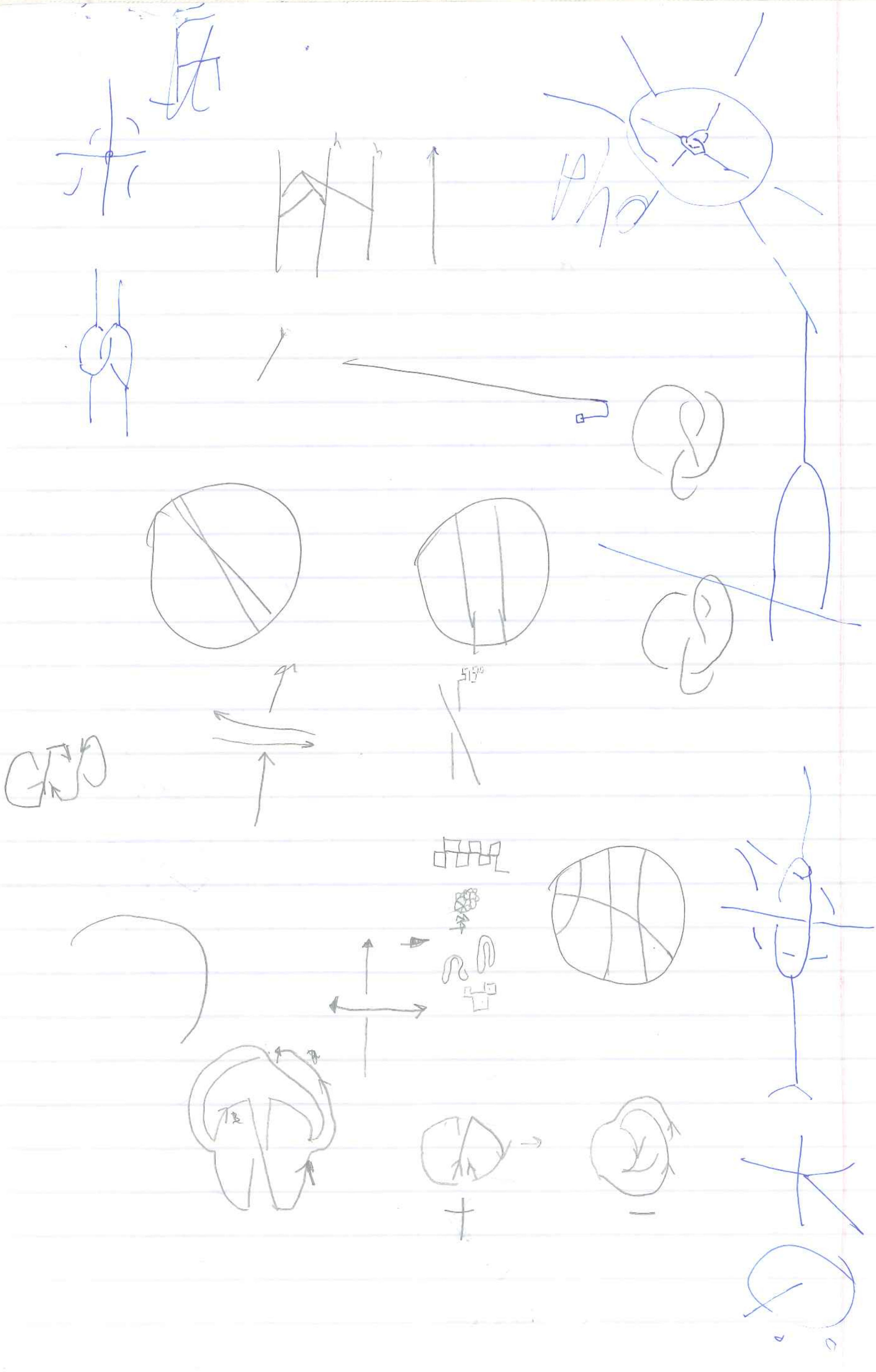
Fluff & Extras
1. Elegant def. of associators.
2. Allows more reductions.
3. ~~Pretty~~ Neat braid version

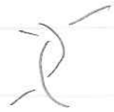
Key Question:

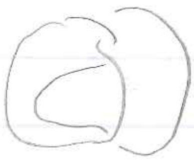
As in slide.

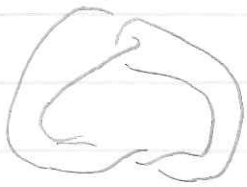
Anti-Associators Front.

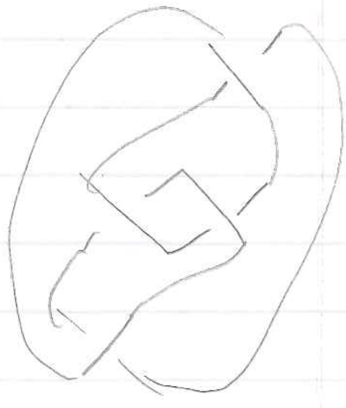
NFAA: National ~~Front Against Associators~~: An organization that I have established to defend knot theory from the tyranny of Associators and to replace them by more humane objects. Our success is not yet complete but we vow to continue fighting

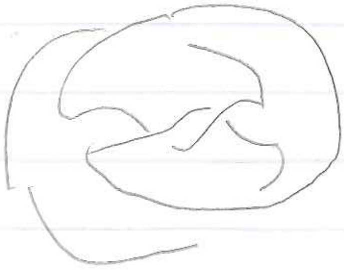


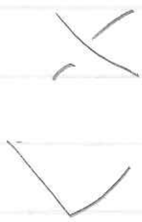


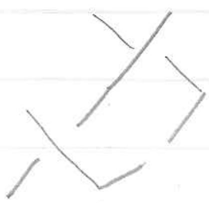












Bracelets and the Goussarov Filtration on the space of knots

Kyoto Sep 25, 2001

The key point: It's fun!

Sad reality: I'll have to start with things you all know about.

0. I follow Goussarov and only touch the presentation.

part I: review

1. brief intro Vass. finite ts: Gouss. finite type:

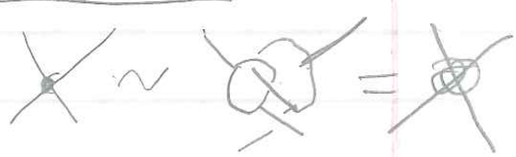
on

Goussarov's thm

$\mathcal{I}_n^V = \mathcal{I}_{2n}^G = \mathcal{I}_{2n+1}^G$

Counter intuitive!

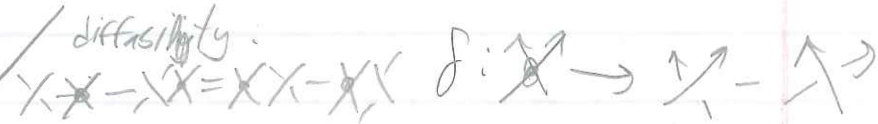
Key point:



board.

$K_{n+1}^V \xrightarrow{\delta_{n+1}} K_n^V \xrightarrow{\delta_n} K_{n-1}^V \to \dots \to K_1^V \to K = K_0^V$

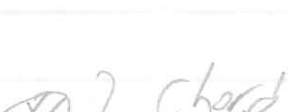
K_n: {knots with n double pts} / diffsibility:



Vassiliev type n: $I|_{\delta^{n+1}K_{n+1}^V} \equiv 0$ $W_I = I \circ \delta^n$

$W_I := I \circ \delta^n : K_n^V / \delta K_n^V \longrightarrow A$

$K_n^V / \mathrm{im}\,\delta = \mathcal{D}_n^{aV} = \{ \otimes \}$ chord diagrams

W descends to $\ker\delta = \langle$  $\rangle$

$K_n^V / \mathrm{im}\,\delta + \ker\delta = \mathcal{A} =$ chord diagrams / 4T & FI

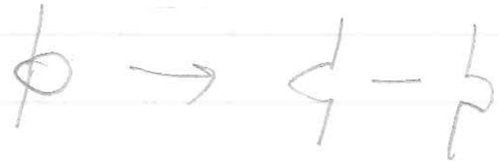
Q: is $\ker\delta_{n+1} = \ker\delta^{n+1}$?
is $\ker\delta = \ker\delta^2$?

Kontsevich: Over $\mathbb{Q}$, $\ker\delta + \mathrm{im}\,\delta = \ker\delta^n + \dots$

part II: What if?

$K^G_{n+1} \to K^G_n \xrightarrow{\delta_n} K^G_{n-1} \cdots \to K^G_0 = K$

$K^G_n = \langle \text{Emb}$  $\rangle$ / ditability

δ: 

What are chord diagrams:

$K^G_n / \delta K^G_{n+1}$ = cyclically ordered links

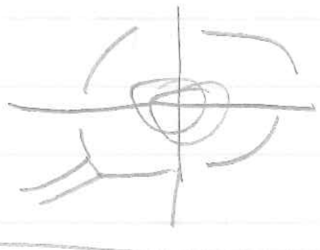
what is ker δ? 1. The two sides of a ring don't see each other (homotopy rel.)

2. Fin() (linearity rel)

$K^G_n / \text{im}\,\delta$ + (part I of ker δ, homotopy + linearity)

= chord diagrams!

part II:

 → 4T!

part III Proof of thm

claim $\delta^{n+1} K^V_{n+1} \overset{1}{\subset} \delta^{2n+1} K^G_{2n+1} \overset{2}{\subset} \delta^{2n+2} K^G_{2n+2}$ <

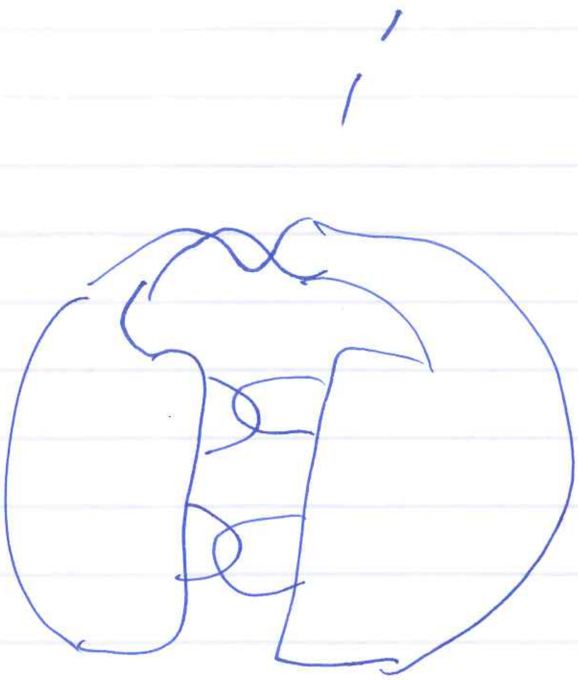
3

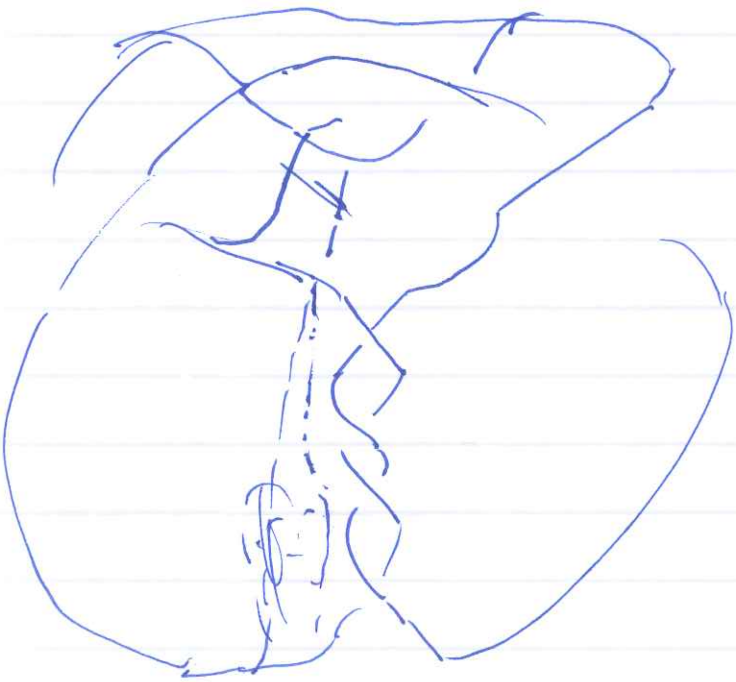
⇑ easy: 

1. easy: 

2. trivial

3. WLOG (modifying by ker δ), the rings of B bound almost disjoint disks.





$\mathbb{R}^1 \hookrightarrow \mathbb{R}^3$ 

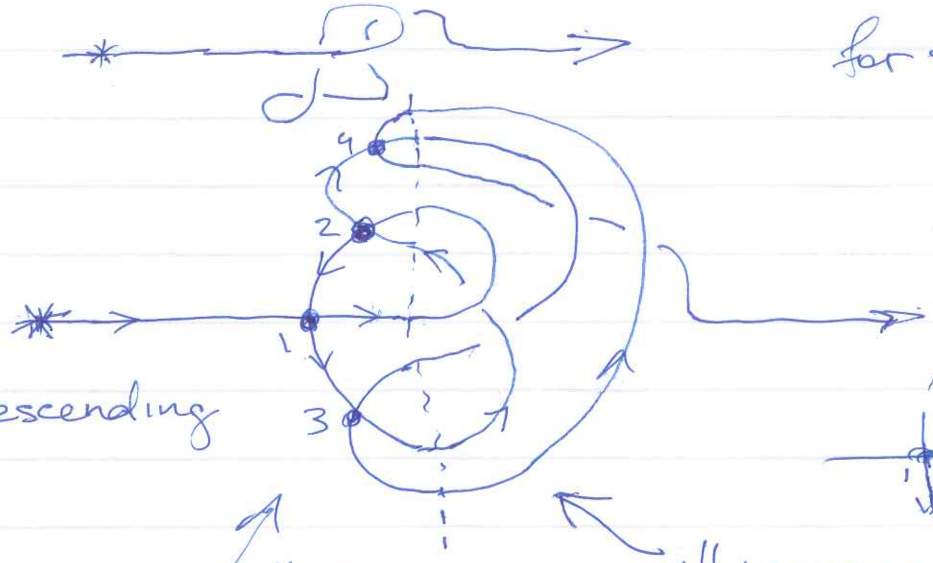
over $\left\{ \begin{array}{l} \mathbb{Z} \\ A \cdot \text{ab. gp} \end{array} \right.$

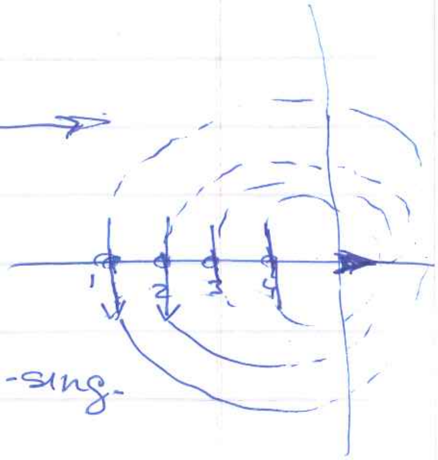
Thm for $\forall$ Vass. inv. of deg $\leq n$ of long knots $\exists$ GD form. with $\leq n$ arrows.

Idea: ① Introduce "unknot" (canonical rep.) for each class of sing. knots – analogue of diag. cross. changes

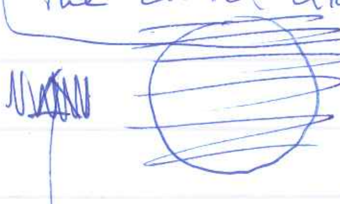
descending diag.

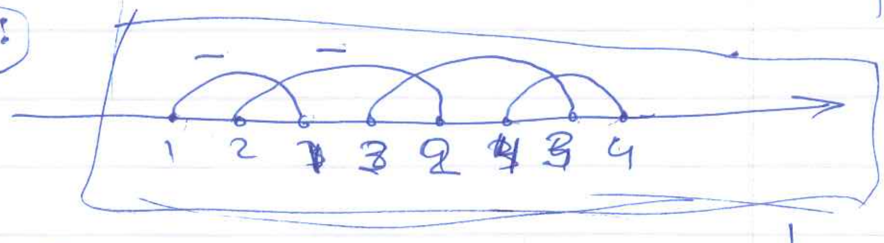
for sing. knots??

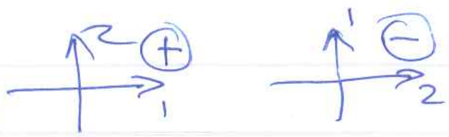


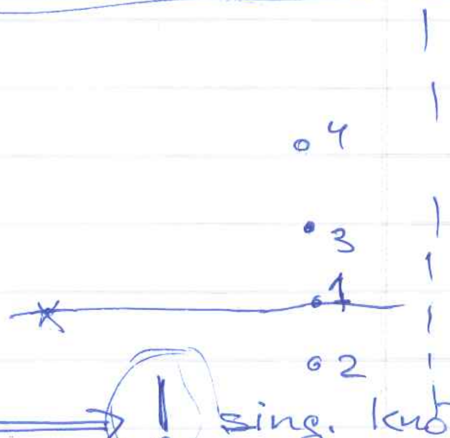


~~sign-decorated~~ the chord diag.!





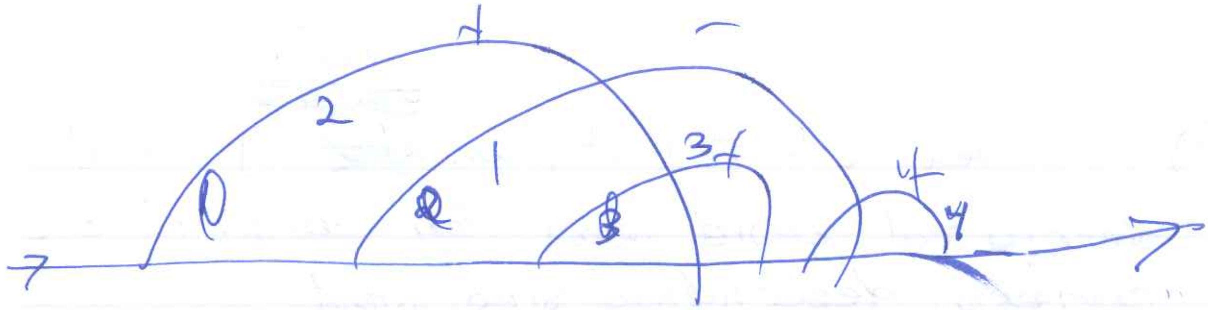


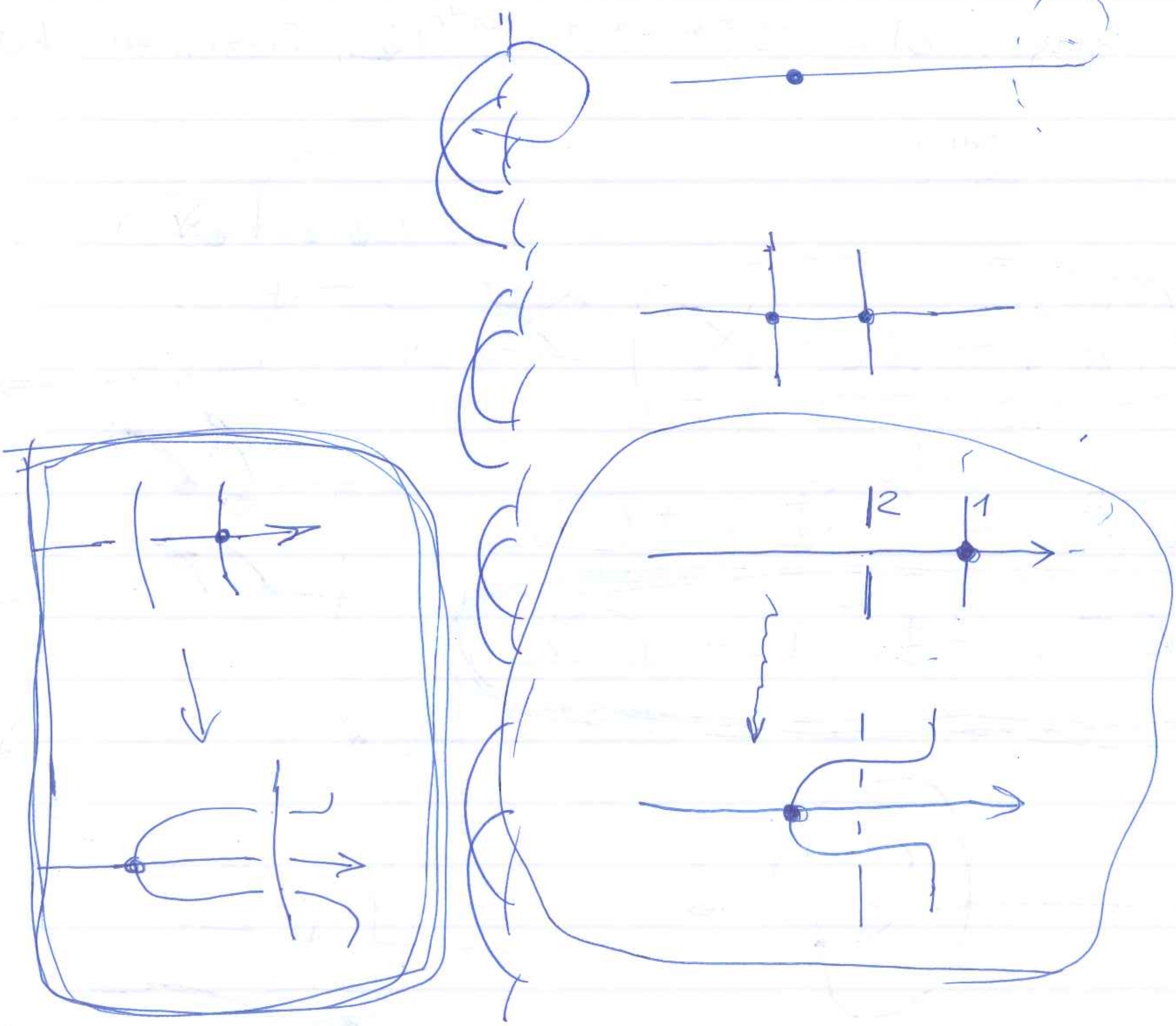


unique desc. diagram

Lemma ??

supp decorated chord diag. $C \Longrightarrow \exists!$ sing. knot s.t. any its desc. diag. has C as the chord diag.



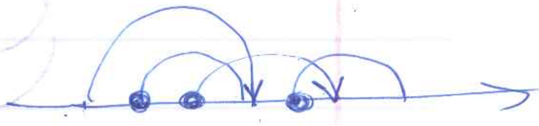


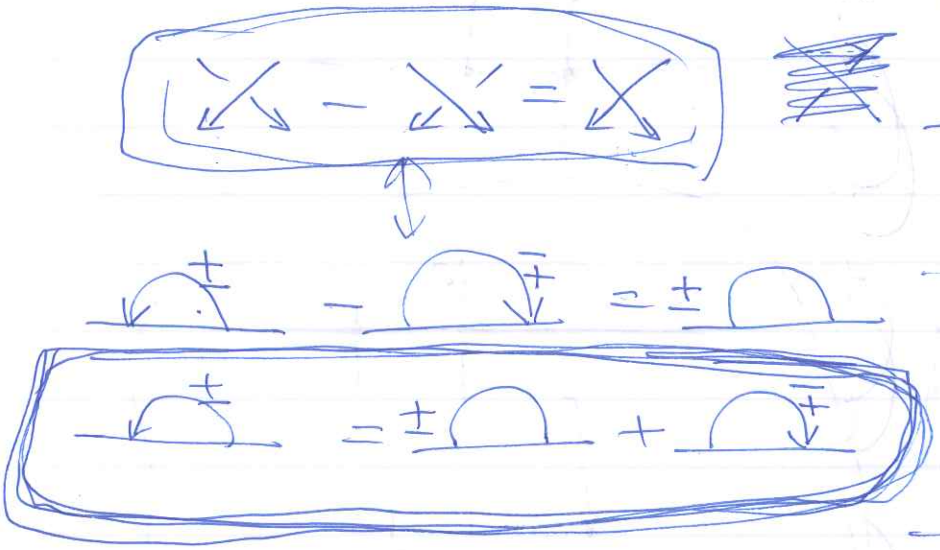
Lemma the representation of a super decorated CD does not depend on the permutation

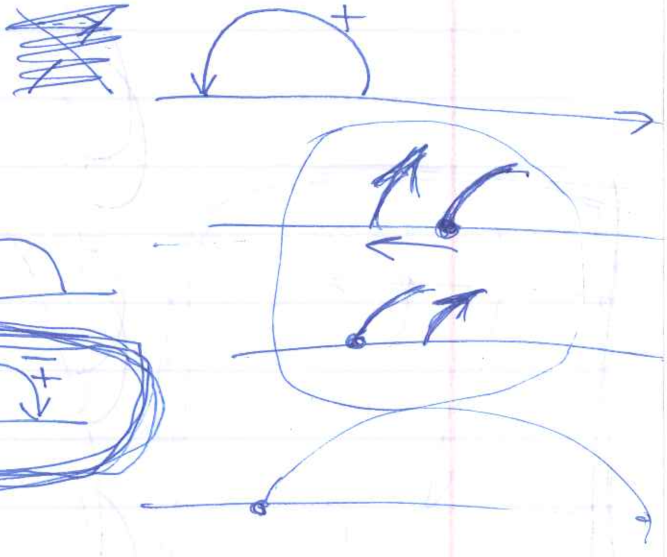
(2) Given a knot invt, we ~~def.~~ ~~compute~~ define it on all decorated chord diag. by comp. it on the corresp. descending sing. knot.

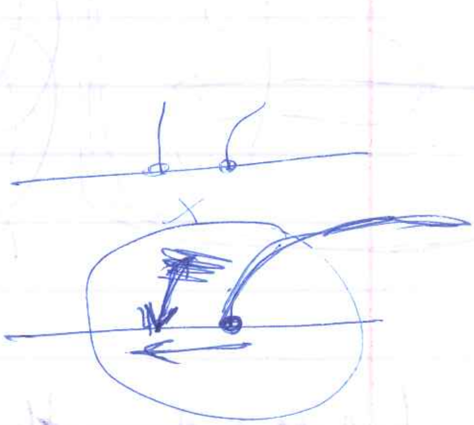
sing. knot $\xrightarrow{\text{cross. ch + isotopy}}$ desc. sing. knot

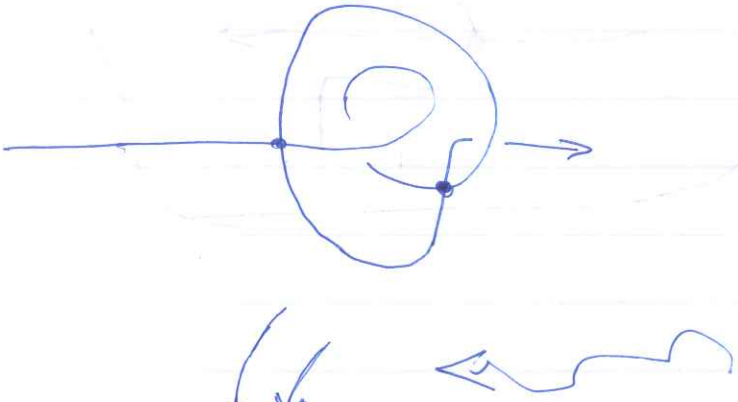
(n-1)-sing.

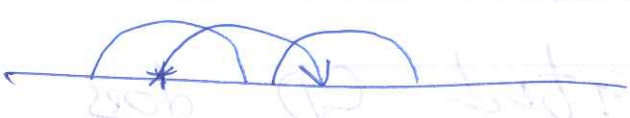


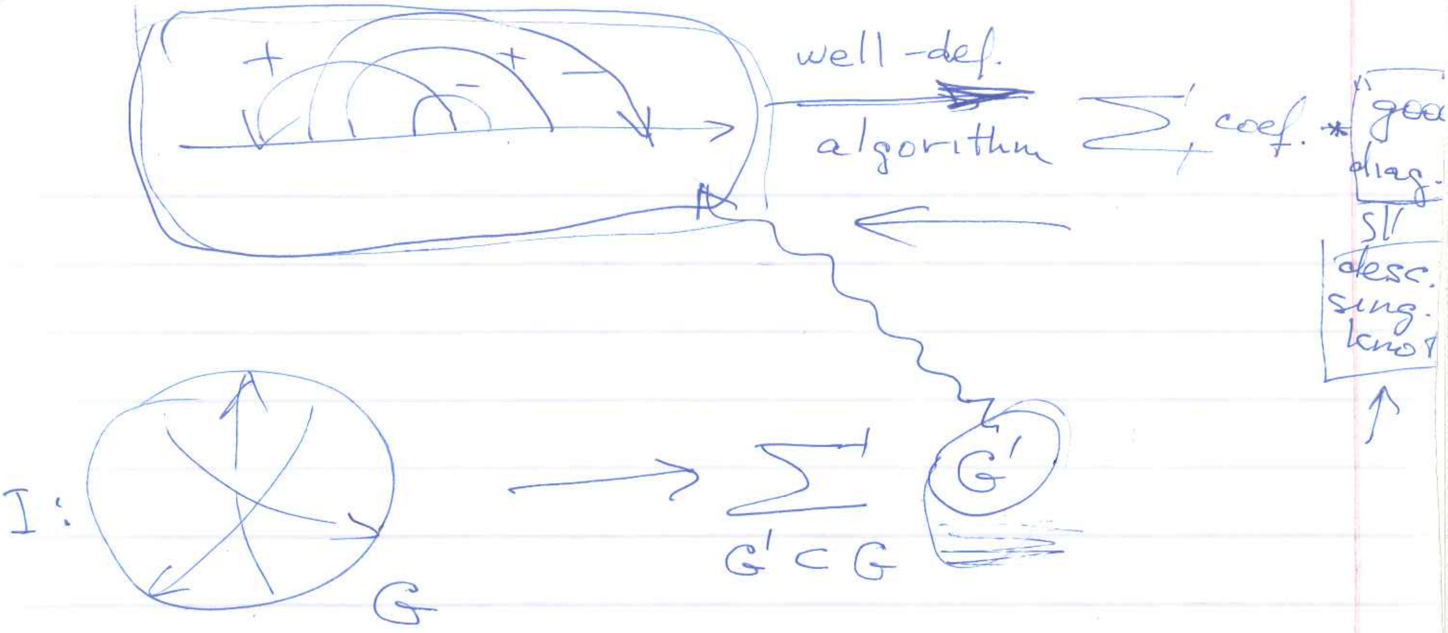


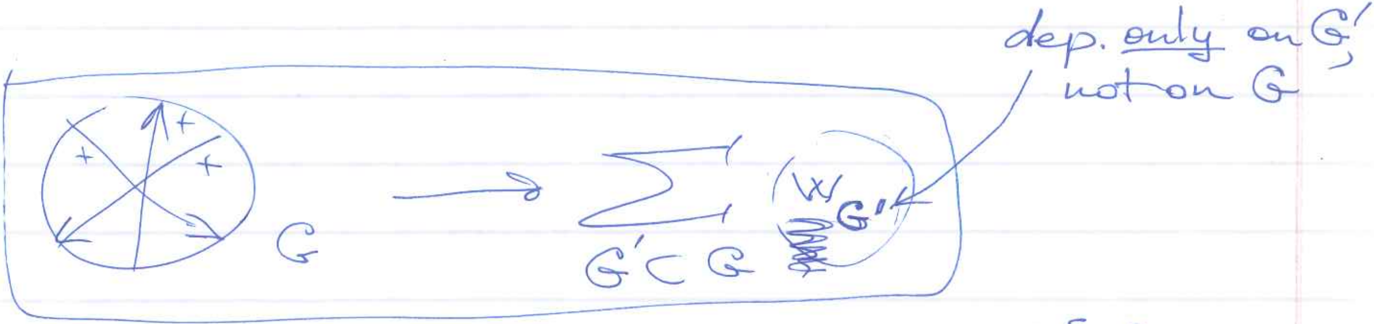


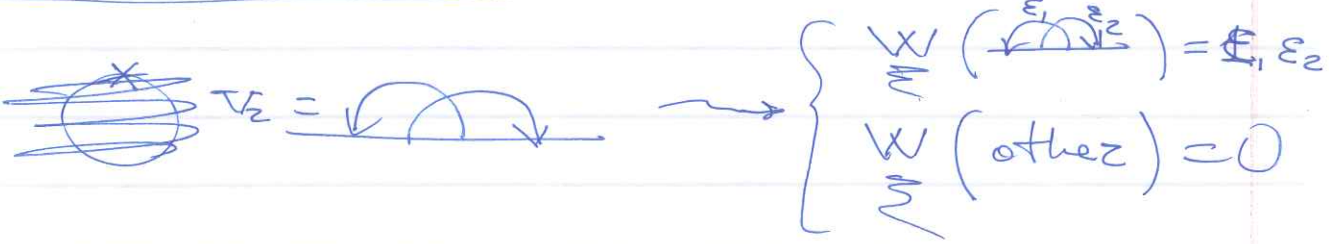












knot invt ⟶ chord diag. using desc. sing. knot

"chords" → chords

value(any desc. diag. of a sing. virtual knot) = same, dep. only on the chord part

$[\![L]\!]$ – a chain complex of graded $\mathbb{Z}$-modules

$$[\![\bigcirc\, L]\!] = (0 \to \underset{\text{height } 0}{V} \to 0) \otimes [\![L]\!] \qquad V = \operatorname{span}\langle v_+, v_- \rangle$$

$\deg v_\pm = \pm 1$, qdim

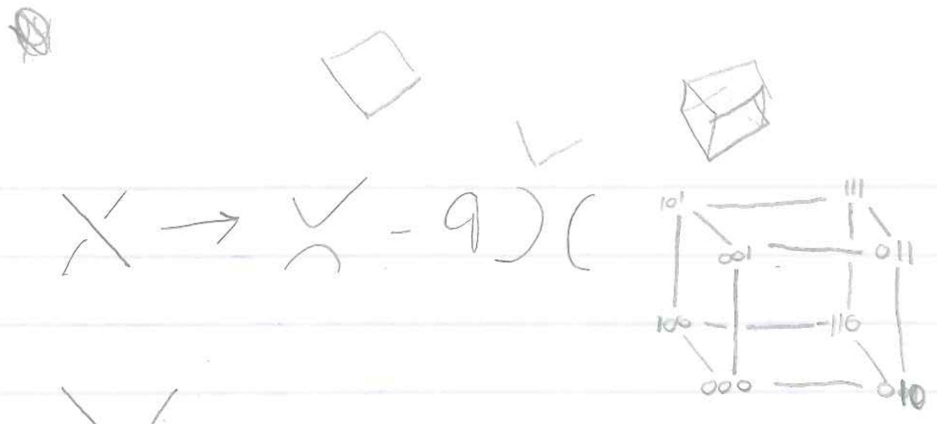
$$\operatorname{qdim} V = q + q^{-1} \qquad \text{with } \operatorname{qdim} \mathcal{O} := \sum q^m \dim \mathcal{O}_m$$

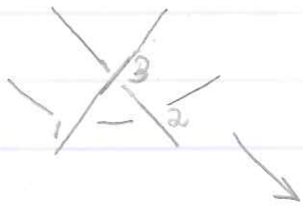
$$\mathcal{O}\{l\}_m = \mathcal{O}_{m-l}$$

$$[\![\times]\!] = \text{Flatten}\Big(0 \to \underset{\text{height } 0}{[\![)(]\!]} \to \underset{\text{height } 1}{[\![\asymp]\!]\{1\}} \to 0\Big)$$

maps $\Bigg\{$

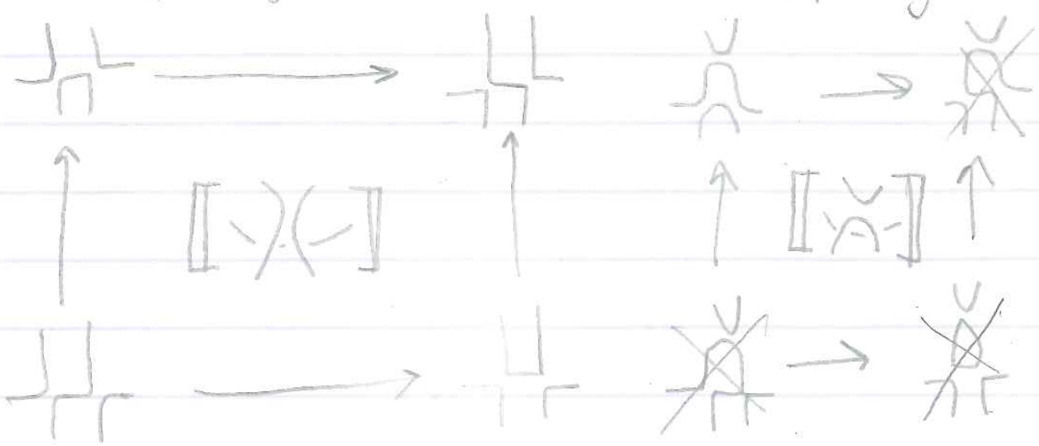
$$\mathcal{H}(L) := H([\![L]\!][-n_-]\{\ldots\}) \qquad Kh_F(L) := \oplus \ldots$$

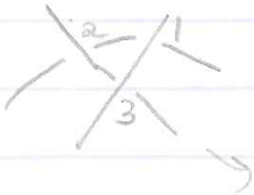




bottom layer

top layer





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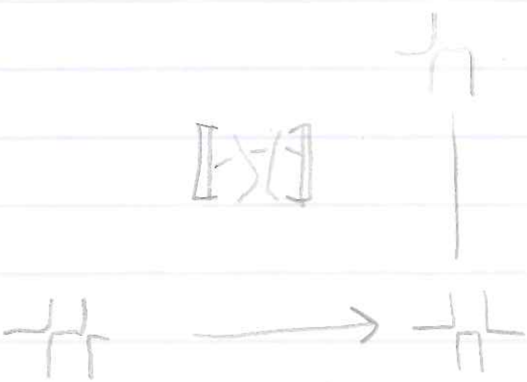
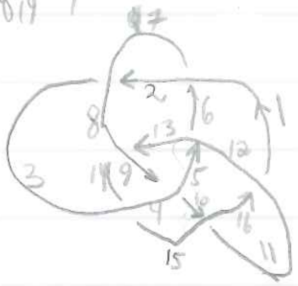
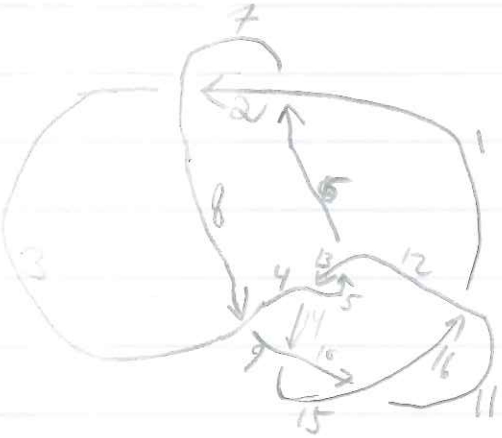
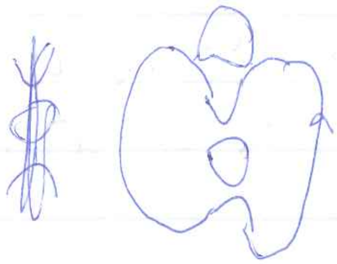


8_{19}-1

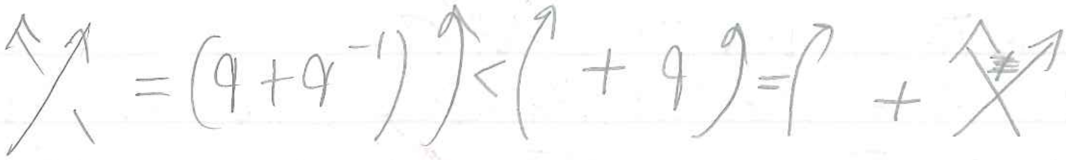


8_{19}-2

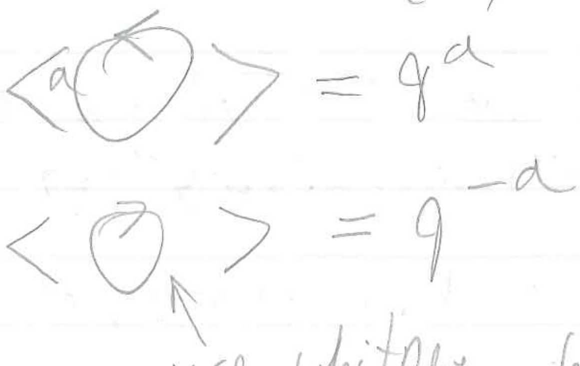




Kauffman Jan 20, 2002:



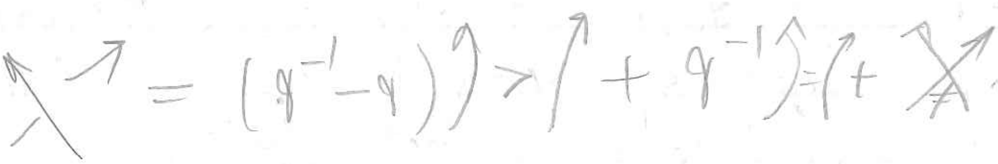
with labels $\{-n, -n+2, \ldots n\}$



use whitney degree

will satisfy

$\langle K_+ \rangle - \langle K_- \rangle = (q - q^{-1}) \langle K_0 \rangle$



Lou Kauffman

BSc MIT 1966

PhD Princeton 1972 W. Browder
"Singularities of complex hypersurfaces & knots"

Since then at UIC on knots & lots of other things

Long term visitor at
Ann Arbor, Berkeley, Saratoga, Bologna, Turin
Recife, Iowa, MSRI, Kyoto, Newton
IHES, Inst Poincaré Paris

Founding Editor of JKTR and of "Knots and everything"

Author of Knots & everything I, "On knots" display
and a few other monographs
& numerous articles

Recipient of 96' Award of Alt. Nat. Phil. Assoc.

Math: Kauffman Bracket } makes Jones child play } Two things named after him
-"- Polynomial } generalizes Jones
Lots of other stuff on state sums, 3 manifolds
& Hopf algebras, knots & DNA ... Lately
"virtual knots", that's today

Style: noted for his style and not just for his math: graphical calculus of everything

Lou Kauffman

Now with Sofia
knots rel DNA.

BSc MIT 1966

PhD Princeton 1972 W. Browder
"Cyclic singularities
of complex hypersurfaces
& knots"

since then at UIC.

Mich Ann Arbor 1976-77

Berkeley many times

Zaragoza	1984
Bologna	1987
Turin	1987
Recife	1988
Iowa	86-87
MSRI	1990
RIMS	1991
Newton	1990, 01
IHES	1988-89
Paris	1997

Recipient of
1993 Warren
McCulloch
of American
Soc. for Cyber.

96' Award of
Am. nat. philosophy
Assoc.

Founding
Editor of JKTR
& Editor of book
series on knots
and everything

math 1981: state sum for Conway poly.
1985: K. Bracket
1985: K. Polynomial
1991: knots, 3folds, Hopf Algebras
1991: virtual knots

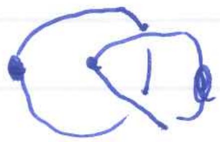
Visiting
SLAC with Noyes
on foundations of
discrete phys.

Slides: 1. Knot theory stinks (picture of an old shoe)
(story of how I tell people what I do)

2. Knot theory is probably dull.
a. slide of Reid. moves
b. slide of Brio knots

4. Flash Lecture's abstract.

5.



$$0 \to A \xrightarrow{x} A \to 0$$

(1) → 0

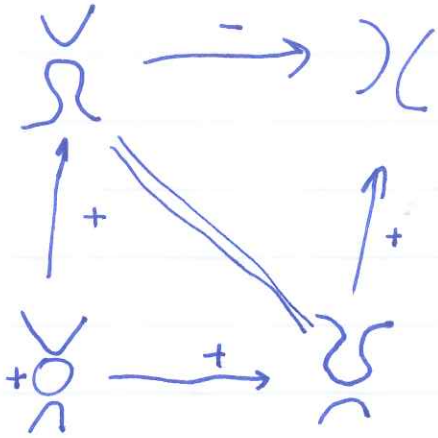
$0 \to$ (X) $\xrightarrow{1} X$

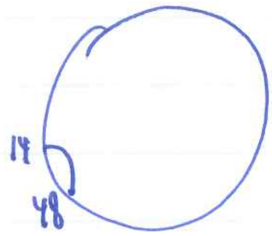
(X) → 0

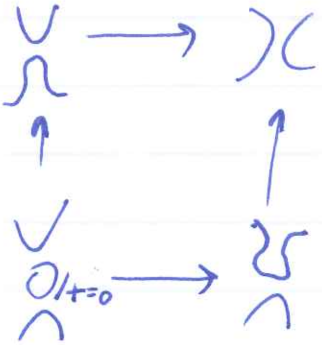
Ker:

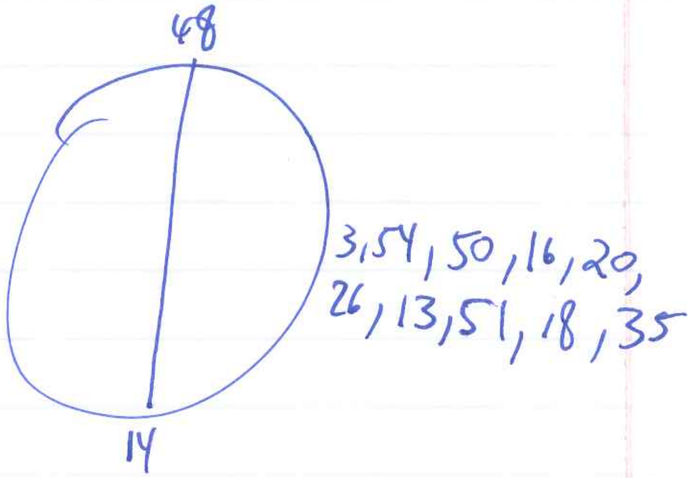
$$0 \to X \xrightarrow{0} X \to 0$$

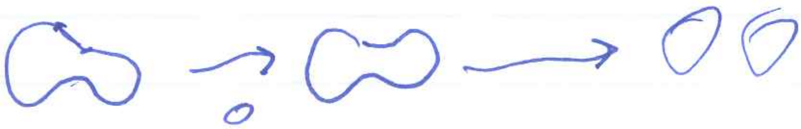
X X





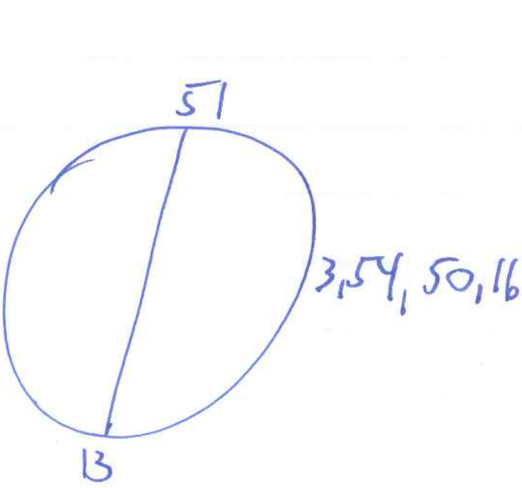


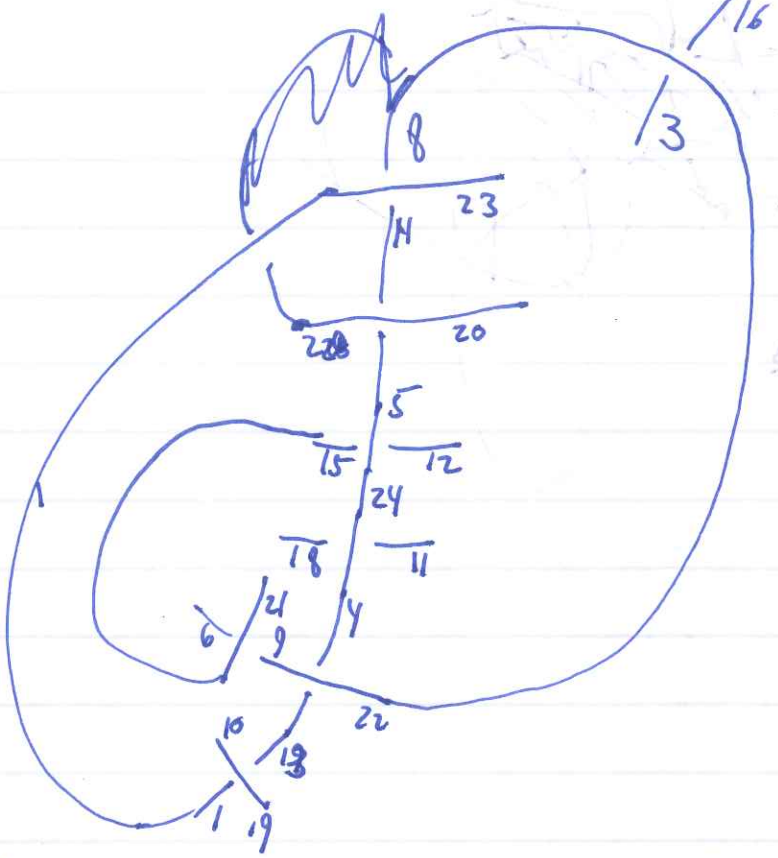




$\begin{pmatrix}2\\1\end{pmatrix}$

+x





16
3
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1
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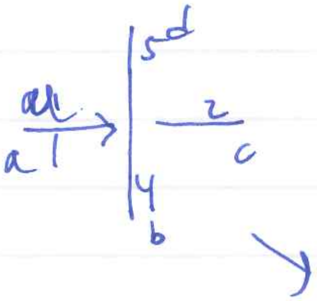
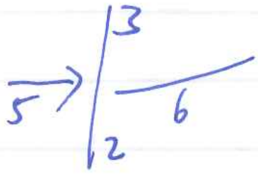
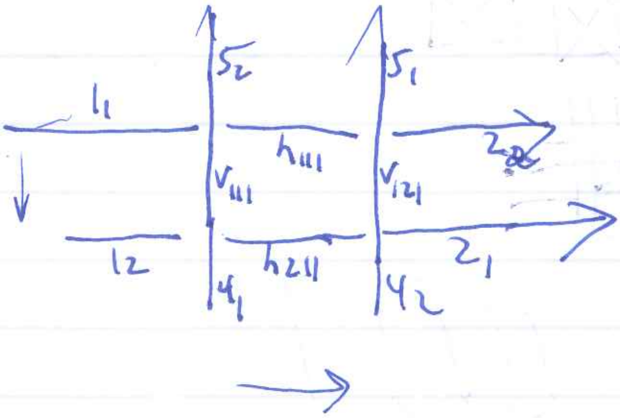


5
d
a 1
2
c
4
b



3
5
6
2



5_2
5_1
l_1
h_{111}
v_{111}
v_{121}
l_2
h_{211}
2_1
4_1
4_2

$v - e + f = 2$

$F = 2 + e - v = 2w$