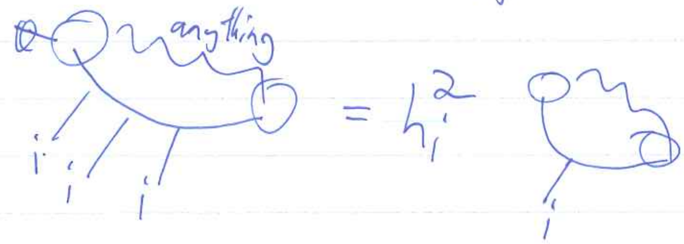
~~$[[x_1 \dots x_n]] - [x_1[x_2 \dots x_n]] - [[x_2 \dots x_n] x_1]$~~

~~$= (n-1)[x_1 \dots x_n] =$~~

Feb 2, 2000 Lieburum

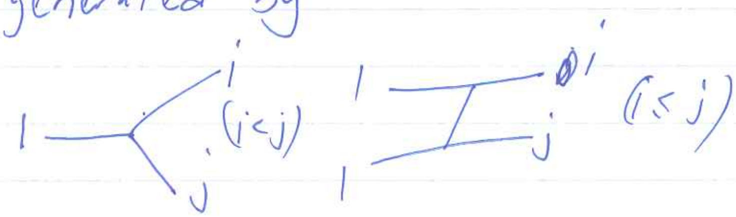
over $Q[h_1^{\pm 1}, \dots, h_n^{\pm 1}]$

rels:
IHX, AS



~~ad H = (ad H)^3~~
$(\mathrm{ad}\, G) = (\mathrm{ad}\, G)^3$
$(\mathrm{ad}\, Q_+) \neq (\mathrm{ad}\, Q_+)^3 = 0$

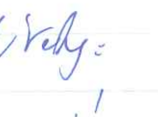
Freely generated by

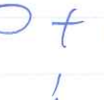


 & struts

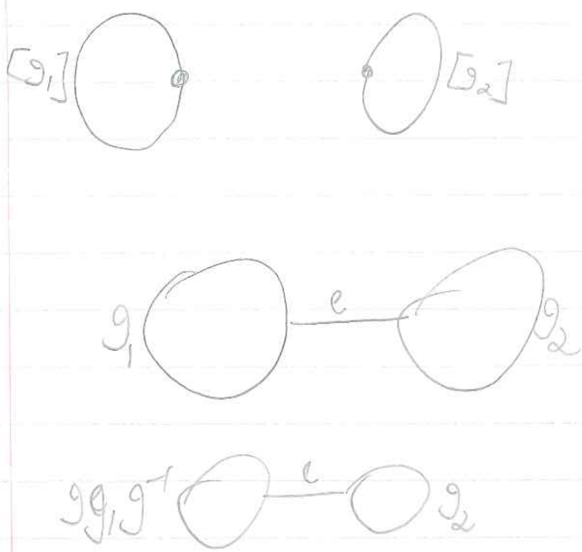
To Do: 1. BCH

2. $\exists$ & can be computed iteratively:

$\phi = \exp(f_1 \bigcirc + f_2$  $)$

3. Compute $\Phi = \exp(f_3$  $)$ in a further quotient of $A(\uparrow\uparrow\uparrow)$

4. compute ϕ



$\beta(x+(x+y+z))$

$\beta(x+y) - \alpha^2 z \cancel{\alpha} + \beta z$

$\beta(x+y+\alpha z) = \beta x + \beta y + (\cancel{-\alpha}\,\alpha^2 + \beta) z$

$-2\beta = -\alpha^2 + \beta$

$-3\beta = -\alpha^2$

$x * y = x + y + [f(x,y)]$

$y\, b[x,z] = z\, b[x,y] + x\, b[y,z]$

$(x*y)*z = (x+y+[f(x,y)]) * z$

$= x+y+[f(x,y)] + z + [f(x+y,z)] +$

$x * y = x + y + f(x,y)\, b[x,y]$

$(x*y)*z = (x+y+f(x,y)\,b[x,y]) * z$

$= x+y+f(x,y)\,b[x,y] + z + f(x+y,z)\,b[x+y+f(x,y)\,b[x,y], z]$

$= x+y+z+ f(x+y,z)\,b[y,z] + f(x+y,z)\,b[x,z]$
$\quad + \left(f(x,y) - z\cdot f(x+y,z)\cdot f(x,y)\right) b[x,y]$

$= x+y+z+\left(f(x+y,z) + \frac{f(x+y,z)-f(x,z)}{y} x\right) b[y,z] + f(x,z)\,b[x,z]$
$\quad + \left(f(x,y) - z\cdot f(x+y,z)\cdot f(x,y) + \frac{f(x+y,z)-f(x,z)}{y} z\right) b[x,y]$

$$y\,b[x,z] = z\,b[x,y] + x\,b[y,z]$$

$$y*z = y+z+f(y,z)\,b[y,z]$$

$$x*(y*z) = x*(y+z+f(y,z)\,b[y,z])$$

$$= x+y+z+f(y,z)\,b[y,z] + f(x,y+z)\,b[x,y+z+f(y,z)b[y,z]]$$

$$= x+y+z+\left(f(y,z) + x f(x,y+z) f(y,z)\right) b[y,z]$$

$$+ f(x,y+z)\,b[x,z]$$

$$+ f(x,y+z)\,b[x,y]$$

$$= x+y+z+f(x,z)\,b[x,z]$$

$$+\left(f(y,z) + x f(x,y+z) f(y,z) + \frac{f(x,y+z)-f(x,z)}{y}x\right) b[y,z]$$

$$+\left(f(x,y+z) + \frac{f(x,y+z)-f(x,z)}{y}z\right) b[x,y]$$

Equations:

Coeff of $b[y,z]$

$$f(x+y,z) + \frac{f(x+y,z)-f(x,z)}{y}x = f(y,z) + x f(x,y+z) f(y,z) + \frac{f(x,y+z)-f(x,z)}{y}x$$

$$f(x+y,z) - f(y,z) = x\left[f(x,y+z)f(y,z) + \frac{1}{y}\left(f(x,y+z) - f(x+y,z)\right)\right]$$

$$f(u,v) = \frac{1}{2} + \beta(u-v) = \frac{1}{2} + \frac{1}{12}(u-v)$$

$$\beta = \alpha^2 - 2\beta \Rightarrow \beta = \frac{1}{12}$$ $\cancel{x}-y-\cancel{z}-\cancel{x}-y+\cancel{z}$

Coeff of $b[x,y]$

$$f(x,y+z) - f(x,y) = z\left[f(x+y,z)f(x,y) + \frac{1}{y}\left(f(x,y+z) - f(x+y,\cancel{z})\right)\right]$$

$$\boxed{\frac{1}{e^x-1} - \frac{1}{x} + 1 = \frac{1}{e^x-1} + \frac{x-1}{x} = f(x,0)}$$

at $z=0$:

$f(x,y) - f(x,y) = 0$ &

$$f(x+y,0) - f(y,0) = x\left[f(x,y)f(y,0) + \frac{1}{y}\left(f(x,y) - f(x+y,0)\right)\right]$$

$\forall\ z=0,\ y=0$

$$f(x,0) - f(0,0) = x\left[f(x,0)f(0,0) + f_2(x,0) - f_1(x,0)\right]$$

Combined equation:

$\cancel{z[f(x+y,z) -}$

$$\Longrightarrow \quad f(x,y) = \frac{xe^x(e^y-1) - y(e^x-1)}{xy(e^{x+y}-1)} = \frac{xe^x + ye^{-y} - (x+y)e^{x-y}}{xy(e^x - e^y)}$$

$$zf(x+y,z)\left(1 - xf(x,y)\right) - zf(y,z) = xf(x,y+z)\left(1 - zf(y,z)\right) - xf(x,y)$$

$$\left(zf(x+y,z) - 1\right)\left(1 - xf(x,y)\right) = \left(xf(x,y+z) - 1\right)\left(1 - zf(y,z)\right)$$

$xe^x(e^y-1) - y(e^x-1)$
$= e^{x+y} - e^x$

$$\left(zf(x+y,z) - 1\right)\left(xf(x,y) - 1\right) = \left(xf(x,y+z) - 1\right)\left(zf(y,z) - 1\right)$$

$$\frac{zf(x+y,z) - 1}{zf(y,z) - 1} = \frac{xf(x,y+z) - 1}{xf(x,y) - 1}$$

$x^3 \cdot y^2 \to (\mathrm{ad}\,x)^3(\mathrm{ad}\,y)^2[x,y]$

at $y=0$

$$\frac{zf(x,z) - 1}{zf(0,z) - 1} = \frac{xf(x,z) - 1}{xf(x,0) - 1}$$

$$\frac{zf(x,z) - 1}{xf(x,z) - 1} = \frac{zf(0,z) - 1}{xf(x,0) - 1} := g$$

$$\frac{1}{k}-\frac{1}{k+1}=\frac{1}{k(k+1)}$$

$$\log e^x e^y = \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \sum_{a_i+b_i>0} \frac{[x^{a_1}y^{b_1} \dots x^{a_k}y^{b_k}]}{(\sum a_i+b_i)\prod a_i!\,b_i!}$$

$$F(x,0) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \sum_{a_i>0} \frac{x^{\sum a_i - 1}}{(\sum a_i + 1)\prod a_i!} \qquad \dots x x^{a} y$$

$$+ \sum_{k=2}^{\infty} \frac{(-1)^k}{k} \sum_{\substack{a_i>0 \\ i=1,\dots,k-1}} \frac{x^{\sum a_i - 1}}{(\sum a_i + 1)\prod a_i!} \qquad \dots x^{a_{k-1}} y^0 x^0 y$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \sum_{\substack{a_i>0 \\ i=1..k}} \frac{x^{\sum a_i - 1}}{(\sum a_i + 1)\prod a_i!} - \sum_{k=1}^{\infty} \frac{(-1)^k}{k+1} \cdot \text{same}$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^k}{k(k+1)} \sum_{\substack{a_i>0 \\ i=1..k}} \frac{1}{x^2} \frac{x^{\sum a_i + 1}}{(\sum a_i + 1)\prod a_i!}$$

$$E = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$$

$$\Longrightarrow \qquad E \cdot x^2 F(x,0) = x \sum_{k=1}^{\infty} \frac{(-1)^k}{k(k+1)} (e^x - 1)^k$$

$$= x + \frac{e^x \cdot x^2}{1-e^x} = x + \frac{x^2 e^x}{1-e^x}$$

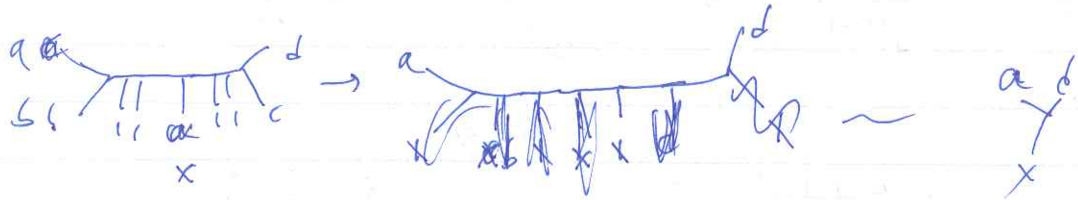
$$\frac{\partial}{\partial x}\left(x^2 F(x,0)\right) = 1 + \frac{x e^x}{1-e^x}$$

$x^2 F(x,0) =$

$$\cancel{x^2 F(x,0) = \log x - \log(e^x - 1) = \log \frac{x}{e^x - 1}}$$

$$F(x,0) = \frac{1}{x^2} \log \frac{x}{e^x - 1}$$

Pumpkin time.



any good?

Haven't looked much yet.

H = deg 3

Y → deg 2

Feb 3, 2000 - BCH mod $[L,[[L,L],[L,L]]]$:

$$x+y+\tfrac{1}{2}[x,y]+[x,fb[x,y]]+[y,gb[x,y]]=\log e^x e^y=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\sum_{a_i+b_i>0}\frac{[x^{a_1}y^{b_1}\cdots x^{a_k}y^{b_k}]}{(\sum a_i+b_i)\prod a_i!\prod b_i!}$$

Computing f: $f=f_1+f_2+f_3$ with:

f_1: $x^{\geq 1}y^{\geq 0}\ldots\ldots x^{\geq 1}y^1 \longrightarrow \displaystyle\sum_{k=2}^{\infty}\frac{(-1)^{k+1}}{k}\sum_{\substack{a_1\geq 1\ b_1\geq 0\\ a_k\geq 1\ b_k=1}}\frac{x^{\sum a_i-2}y^{\sum b_i-1}}{(\sum a_i+b_i)\prod a_i!\prod b_i!}$

f_2: $x^{\geq 1}y^{\geq 0}\ldots x^{\geq 1}y^0x^0y^1 \longrightarrow$

etc.

f_3: $-x^{\geq 1}y^{\geq 0}\ldots x^{\geq 0}y^{\geq 1}x^1y^0 \longrightarrow$

$F_1: x^{\alpha+1}y^{\beta}$

$$E(x^2yF_1)=\sum_{l=0}^{\infty}\frac{(-1)^{l+1}}{l+2}\sum_{\substack{\alpha,\beta,\gamma\geq 0\\ a_j+b_j\geq 1}}\frac{x^{\alpha+1}y^{\beta}x^{\sum a_j}y^{\sum b_j}x^{\gamma+1}}{(\alpha+1)!\,\beta!\,(\gamma+1)!\,\prod a_j!\,b_j!}$$

$$=\sum_{l=0}^{\infty}\frac{(-1)^{l+1}}{l+2}(e^x-1)^2e^y(e^xe^y-1)^l$$

F_2: $x^{1+\alpha}y^{\beta}\ldots x^{1+\gamma}y^0x^0y^1$:

$$E(x^2yF_2)=\sum_{l=0}^{\infty}\frac{(-1)^{l}}{l+3}\sum_{\substack{\alpha,\beta,\gamma\geq 0\\ a_j+b_j\geq 1}}\frac{x^{1+\alpha}y^{\beta}x^{1+\gamma}x^{\sum a_j}y^{\sum b_j}}{(1+\alpha)!\,\beta!\,(1+\gamma)!\,\prod a_j!\,b_j!}y$$

$$=\sum_{l=0}^{\infty}\frac{(-1)^{l}}{l+3}(e^x-1)^2e^y(e^xe^y-1)^l\cdot y$$

$$E(x^2yF_3)=\sum_{l=0}^{\infty}\frac{(-1)^{l+1}}{l+3}(e^x-1)e^{x+y}(e^y-1)x(e^xe^y-1)^l$$

Claim $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+n+1}}{k+n} t^{k} = \frac{1}{t^n}\left(\log(1+t) - t + \frac{t^2}{2} - \frac{t^3}{3} \cdots \pm \frac{t^n}{n}\right)$

~~Thus~~

$$\cancel{E(x^2yF) = (e^x-1)^2 e^y\left(\frac{1}{(e^{x+y}-1)^2}\left(x+y-(e^{x+y}-1)+\frac{(e^{x+y}-1)^2}{2}\right) - y/2\right)}$$

$(n \geq 1)$

Claim $(n \geq 1)$ $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k+n+1}}{k+n} t^{k} = \sum_{k=n}^{\infty} \frac{(-1)^{1+k}}{k} t^{k-n} = \frac{1}{t^n}\left(\log(1+t) - t + \cdots \pm \frac{t^{n-1}}{n-1}\right)$

$$= \frac{1}{t^n}\left(\log(1+t) - t + \frac{t^2}{2} - \cdots \pm \frac{t^{n-1}}{n-1}\right)$$

$$E(x^2yF) = \frac{(e^x-1)^2 e^y}{(e^{x+y}-1)^2}\left(x+y-(e^{x+y}-1)\right)$$

$$+ \frac{(e^x-1)^2 e^y y}{(e^{x+y}-1)^3}\left(x+y-(e^{x+y}-1)+\frac{(e^{x+y}-1)^2}{2}\right)$$

$$- \frac{(e^x-1)(e^y-1)e^{x+y}\cdot x}{(e^{x+y}-1)^3}\left(x+y-(e^{x+y}-1)+\frac{(e^{x+y}-1)^2}{2}\right)$$

$$=$$

BCH mod $[[L,L],[L,L]]$:

$$x+y+Fb[x,y] = \log e^x e^y = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sum_{a_i+b_i>0} \frac{[x^{a_1}y^{b_1}\cdots x^{a_k}y^{b_k}]}{(\sum a_i+b_i)\prod a_i!\prod b_i!}$$

$\cancel{x^{a_1}y^{b_1}\cdots x^{1+\alpha}y^1}$

$E(xyF) =$

$$\sum_{l=0}^{\infty} \frac{(-1)^{l}}{l+1}(e^x-1)\cdot y(e^{x+y}-1)^l \qquad x^{a_1}y^{b_1}\cdots x^{1+\alpha}y^1$$

$$+\sum_{l=0}^{\infty} \frac{(-1)^{l+1}}{l+2}(e^x-1)\cdot y(e^{x+y}-1)^l \qquad x^{a_1}y^{b_1}\cdots x^{1+\alpha}y^0x^0y^1$$

$$-\sum_{l=0}^{\infty} \frac{(-1)^{l+1}}{l+2}\, x(e^y-1)(e^{x+y}-1)^l e^x \qquad -x^{a_1}y^{b_1}\cdots x^{\beta}y^{1+\alpha}x^1y^0$$

$$\boxed{\text{Lemma: } \sum_{l=0}^{\infty}\frac{(-1)^{l+n+1}}{l+n}t^l = \sum_{l=n}^{\infty}\frac{(-1)^{l+1}}{l}t^{l-n} = t^{-n}\left(\log(1+t)-t+\frac{t^2}{2}\cdots\pm\frac{t^{n-1}}{n-1}\right)}$$

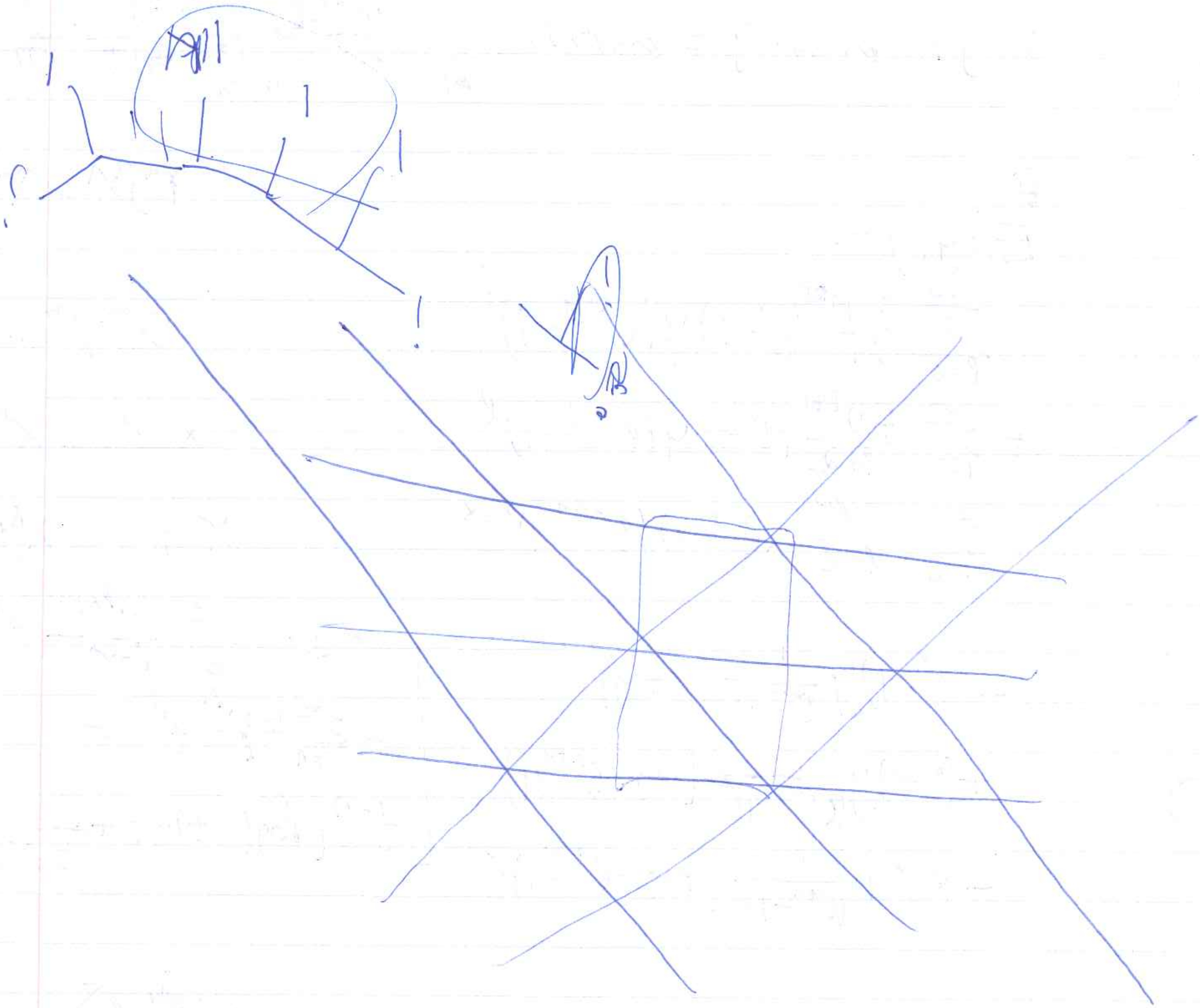
$$=(e^x-1)y\frac{1}{e^{x+y}-1}(x+y)$$

$$+(e^x-1)y\frac{1}{(e^{x+y}-1)^2}(x+y-e^{x+y}+1)$$

$$-x(e^y-1)\frac{1}{(e^{x+y}-1)^2}(x+y-e^{x+y}+1)e^x$$

$$F' = \frac{xe^x+ye^{-y}-(x+y)e^{x-y}}{xy(e^x-e^{-y})} = \frac{xe^{x+y}+y-(x+y)e^x}{xy(e^{x+y}-1)} \quad \cancel{e^{x+y}-1}$$

$$E(xyF') = \frac{x(e^{x+y}+xe^{x+y}-e^x-(x+y)e^x)(e^{x+y}-1)-(x+y)e^{x+y}(xe^{x+y}+y-(x+y)e^x)+y(xe^{x+y}+1-e^x)(e^{x+y}-1)}{(e^{x+y}-1)^2}$$



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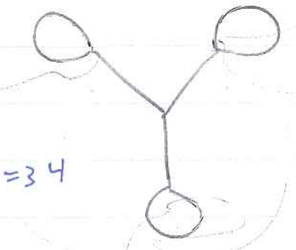
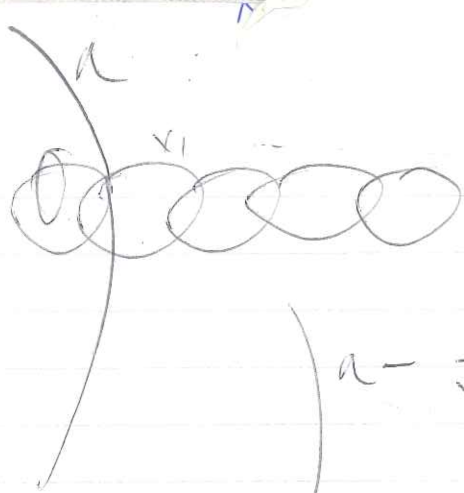


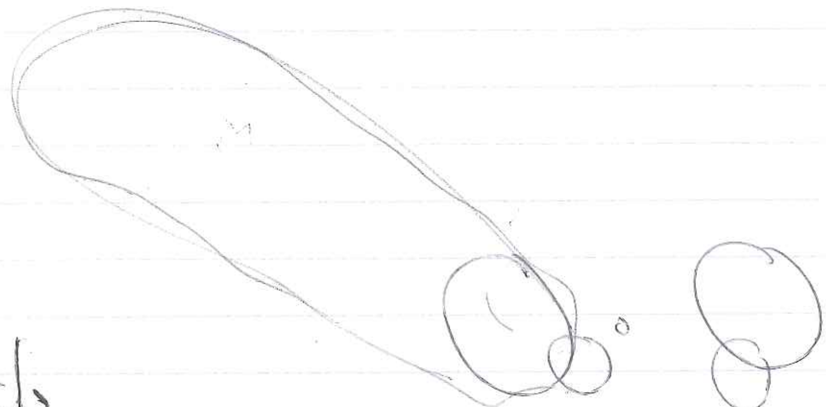
$5+11+5+3+10=34$

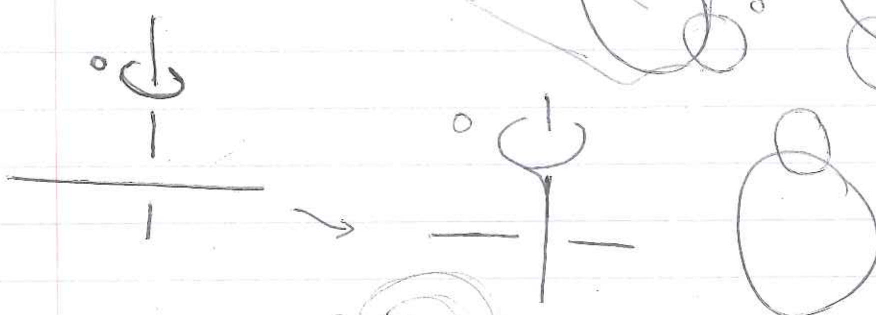


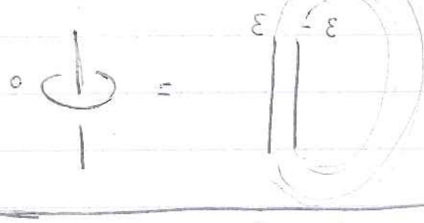
$(A^{-1})^{ij} = lk(\mu_i, \mu_j) \in \mathbb{Q}/\mathbb{Z}$

$$a - \cfrac{1}{x_1 - \cfrac{1}{\ddots}}$$

$$a - \frac{1}{0} = \infty$$





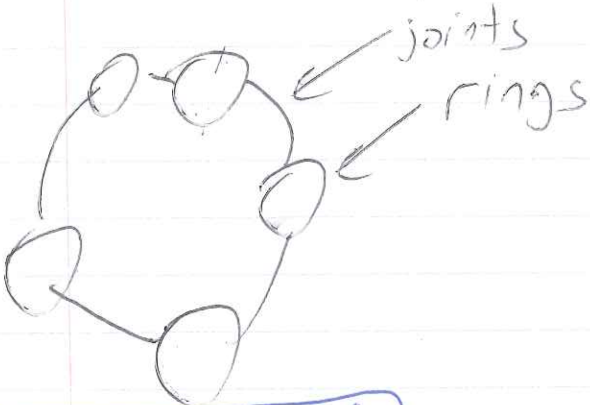


If M^3, N^3 have same homology + lk form then they can be obtained one from another by sequences of Y-moves.

$M^3 = S^3_L$

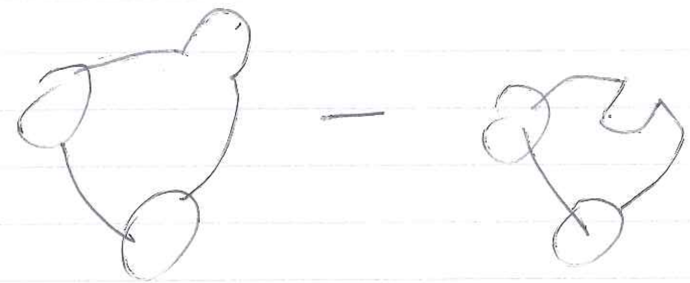
$N^3 = S^3_{L'}$

5-bracelent



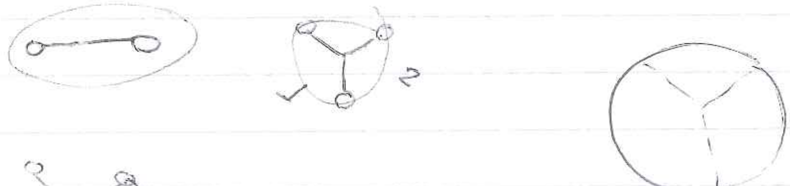
$\delta : (n+1)$ bracelets $\to$ n bracelets

Claim: V-moves connect all manifolds w/ fixed homology & linking form
PF Matveev, Generalized Surgery of Three dimensional - - - -

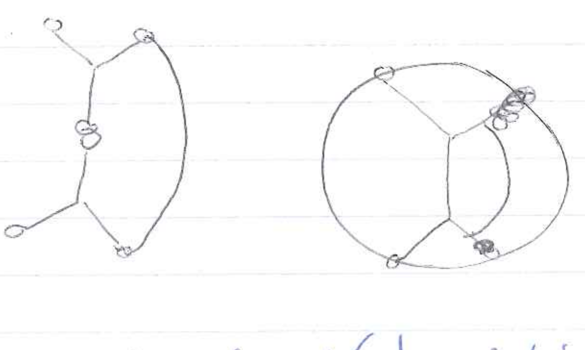


$$\text{chord-diagrams} \stackrel{?}{=} K_n / \delta K_{n+1}$$
$$= \text{cyclic links}$$

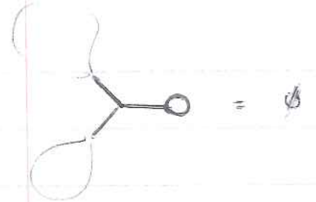
4T: . - - -

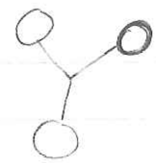


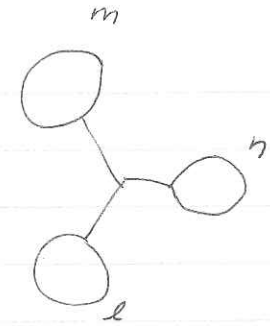
Δ - deg 2

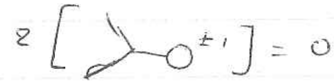


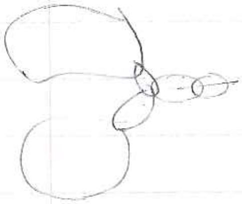
Murakami–Nakanishi "On a certain move generating link-homology" Math Ann. 284 (1989) 75-9

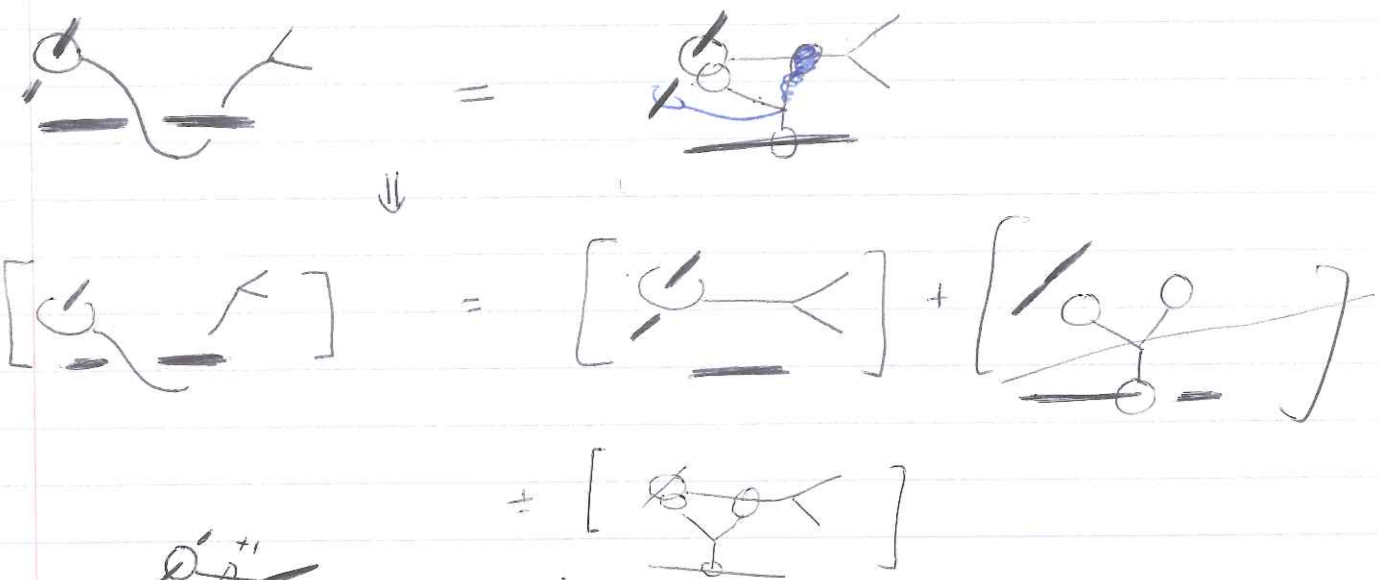


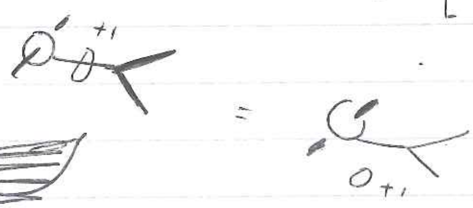


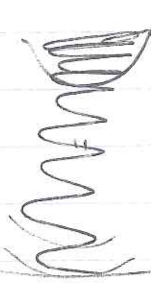




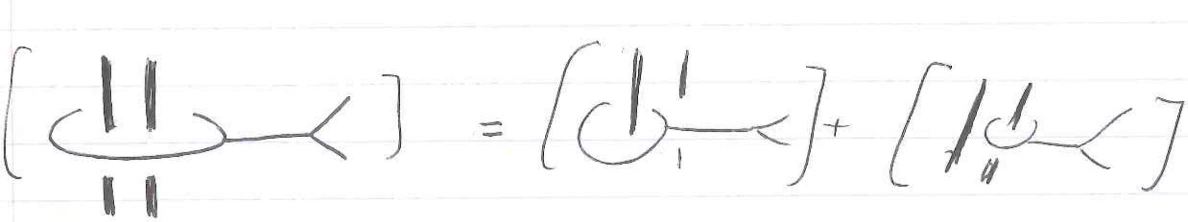


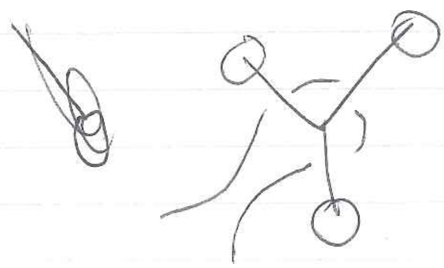




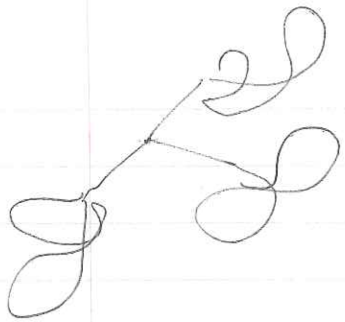


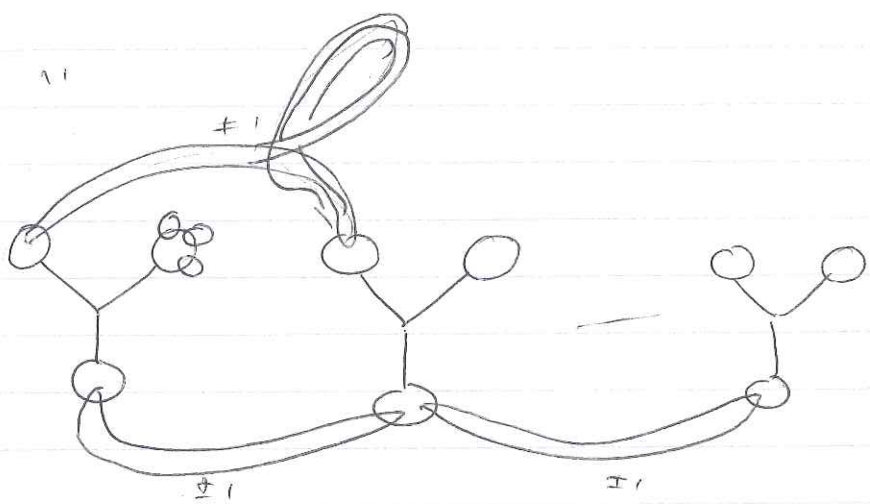
$[x, xy] = [x, y]$

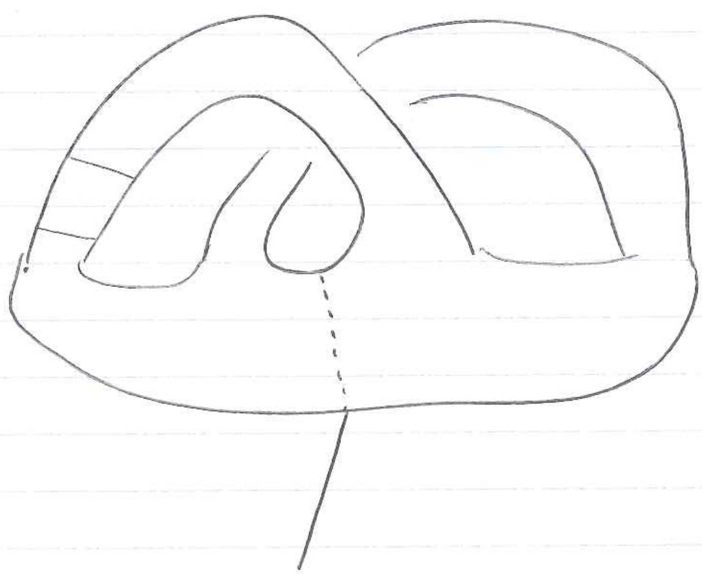


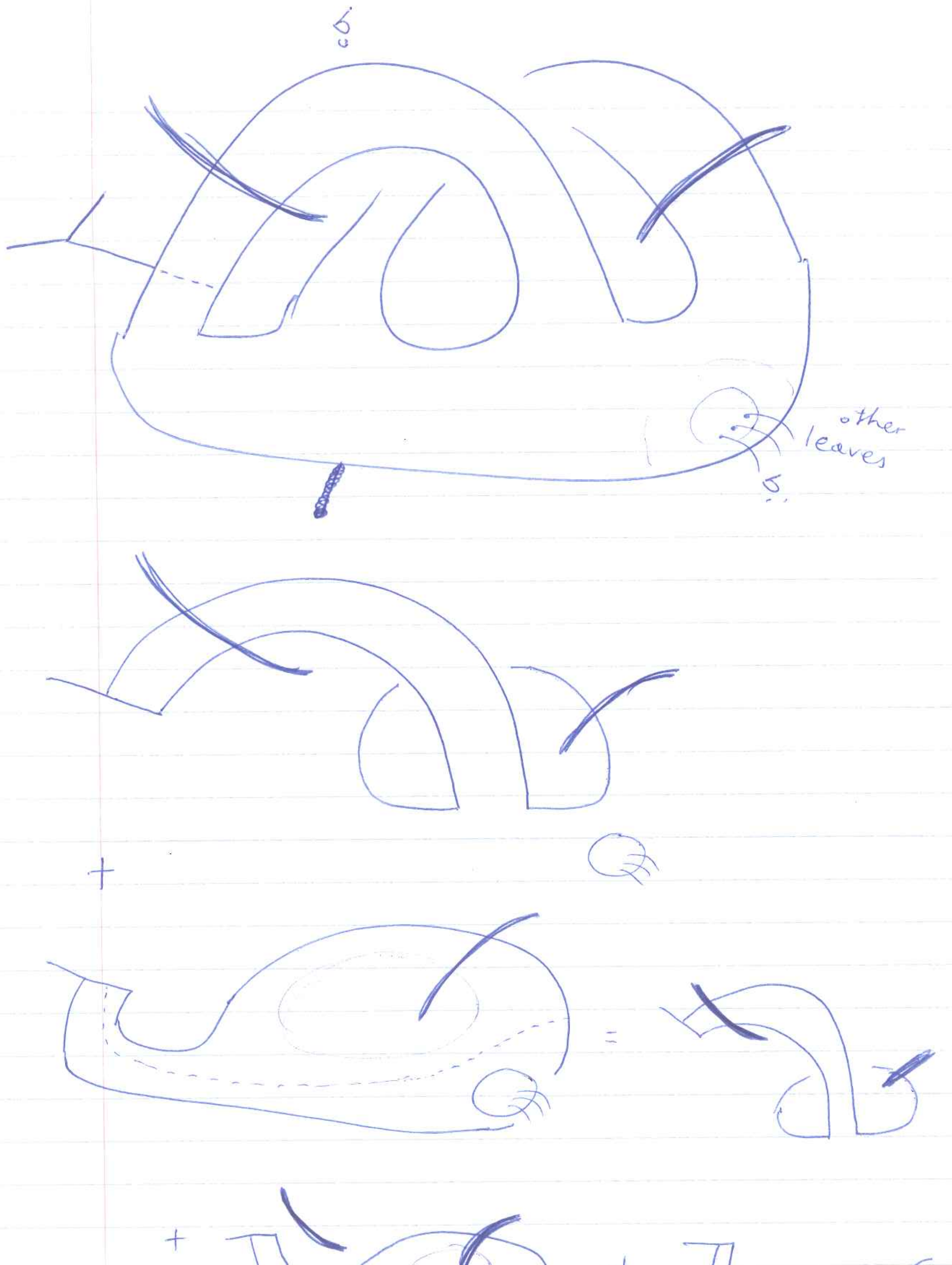


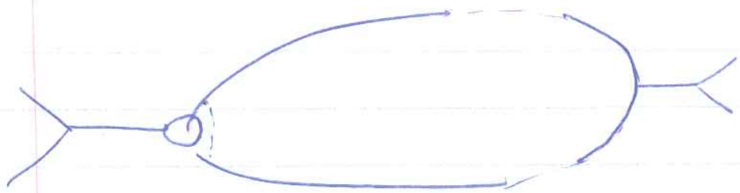
$[x, yz] = [x, y] \cdot [x, z]$

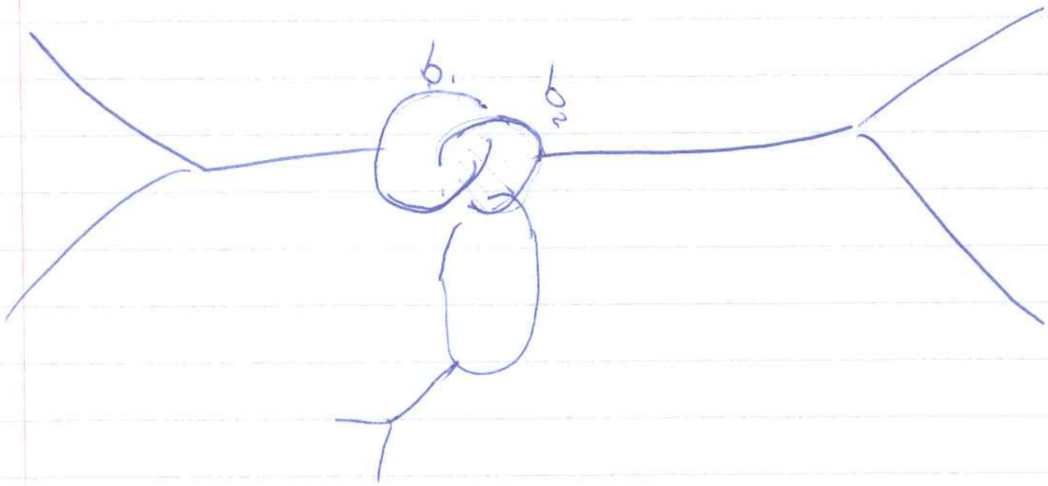






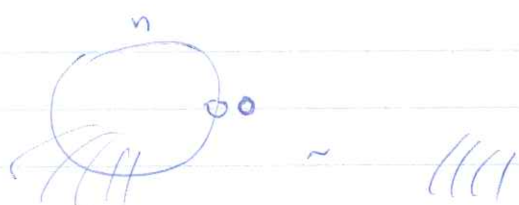

other leaves





Suppose M, N have <u>same</u> <u>homo + lk form</u>.

$M = S^3_L$
$N = S^3_{L'}$ and $lk(L)$ $lk(L')$ are S-equivalent.



$= S^3$

$$A \sim \left(\begin{array}{c|cc} A & \begin{matrix}\times\\ \times\end{matrix} & 0 \\ \hline \times\times\times & * & 1 \\ 0 & 1 & 0 \end{array}\right)$$

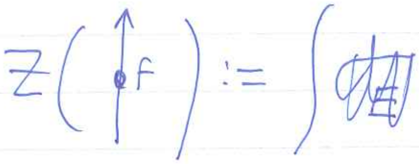
By doing Kirby moves on L or L' we can <u>assume</u>

$$\boxed{lk(L) = lk(L')}$$

<u>Claim:</u> If $lk(L) = lk(L')$ then L and L' are Δ-equiv.

Jan 4, 2000

$$F = \frac{p}{q} = \langle a_1 \dots a_\ell \rangle = -\frac{1}{a_1 - \frac{1}{\ddots}}$$



$G = K$ compact Lie gp

$$X_G = G_\Delta \backslash G^3 / G_\Delta = \{(a,b,c) : abc = 1\}/\text{conj}$$

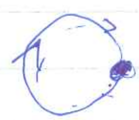
$$(f,g,h) \longmapsto (fg^{-1}, gh^{-1}, hf^{-1})$$

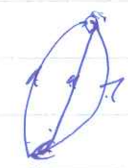
$$X_\mathfrak{g} = \{(A,B,C) \in \mathfrak{g}^3 : A+B+C=0\}/\text{adjoint action}$$

$$\begin{array}{cc} X_G & X_\mathfrak{g} \\ \downarrow & \downarrow \pi \\ & (\mathfrak{g}/G)^3 = (\mathfrak{t}^*_+)^3 \end{array}$$

$$(U(\mathfrak{g})^{\mathfrak{g}})^* \sim (\mathcal{I} \ldots)^*$$

$$(\mathrm{Sym}(\mathfrak{g}))^{G} \simeq S(\mathfrak{h})^W$$





$$\text{RHS} \longrightarrow \bigoplus_{\lambda,\mu,\nu} \mathrm{End}\left((V_\lambda \otimes V_\mu \otimes V_\nu)^G\right)$$

fibers of π:

$$\pi^{-1}(\lambda,\mu,\nu) \underset{\in (\mathfrak{t}^*_+)_\mathbb{Z}}{} \quad \text{projective algebraic} = \mathrm{Proj} \bigoplus_{k \in \mathbb{N}} (V_{k\lambda} \otimes V_{k\mu} \otimes V_{k\nu})^G$$

Cartan product: $V_\lambda \otimes V_\mu \twoheadrightarrow V_{\lambda+\mu}$

comes with a natural line bundle $\mathcal{L}$

$$\Gamma(\mathcal{L}^k) = (V_{k\lambda} \otimes V_{k\mu} \otimes V_{k\nu})^G$$

Gorelick, Feb 2, 2000

$\mathfrak{g}$: $\mathbb{C}$ SS Lie Alg $\quad \mathfrak{g} = \mathfrak{n}^- \oplus \mathfrak{h} \oplus \mathfrak{n}^+$

Separation thm: (Kostant) $\quad H \subset_{ad\,\mathfrak{g}} U(\mathfrak{g})$

$$H \otimes Z(\mathfrak{g}) \cong U(\mathfrak{g})$$

$$H = \bigoplus_{V \in Irr} V^{\dim V^{\mathfrak{h}}}$$

Ann. Theorem: (Duflo)

$$\operatorname{Ann} M = U(\mathfrak{g}) \operatorname{Ann}_{Z(\mathfrak{g})} M$$

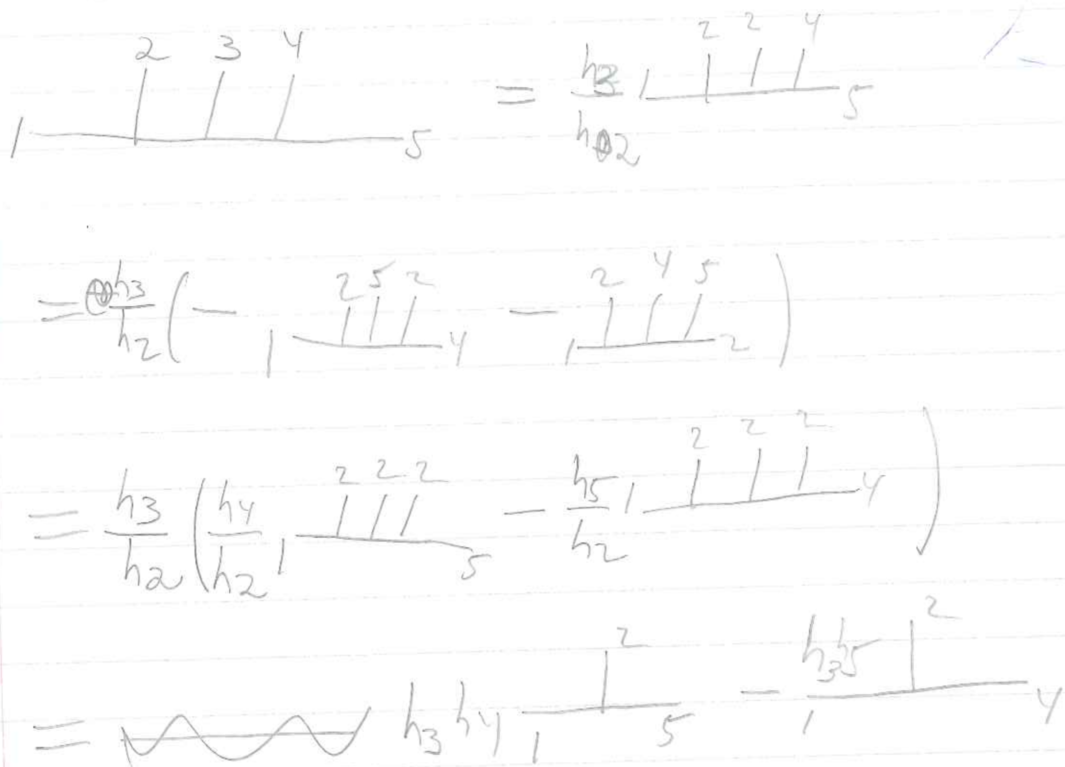
M = Verma Mod.

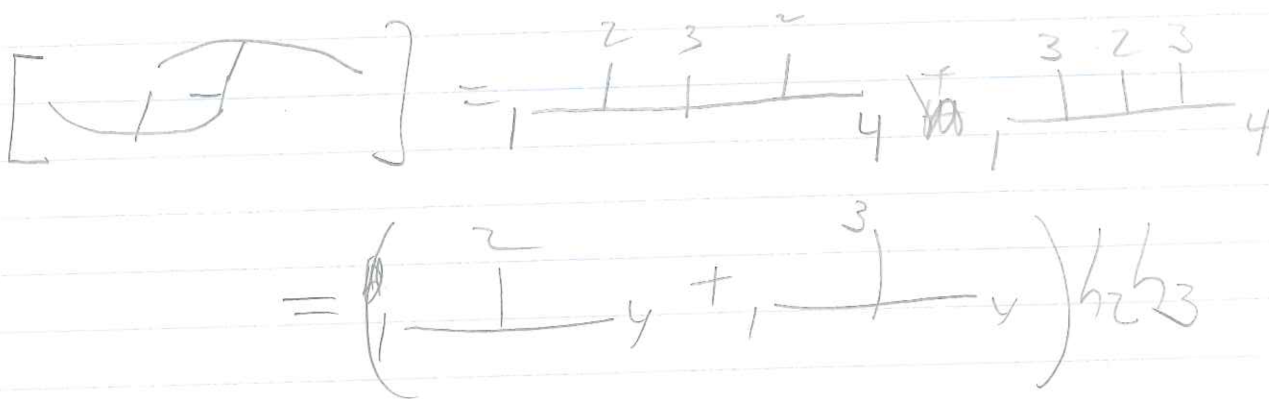
$$M(\lambda) = U(\mathfrak{g}) \otimes_{U(\mathfrak{h}+\mathfrak{n}^+)} \mathbb{C}_\lambda \qquad \lambda \in \mathfrak{h}^*$$

$\dim \operatorname{Hom}_{\mathfrak{g}} (V, H) = \dim V^{\mathfrak{h}} \qquad V \in Irr$

(# of times $U(\mathfrak{g})$ contains rep. V)

$\Lambda^2(\Lambda^3 V) =$





$$\Lambda^4(V) \longrightarrow S(V)\otimes\Lambda^3(V)$$

$\mu(f) = \int f' \, d\tilde{\mu} \qquad \nu(f) = \int f' \, d\tilde{\nu}$

$$(\mu * \nu)(f) = (\mu \otimes \nu)(f(x+y))$$

$$= \int \left(f_\mu(x+y) \right)_y d\tilde{\nu} \Big)_x d\tilde{\mu}$$

$$= \int \left(\int f'(x+y) \, d\tilde{\nu} \right)_x d\tilde{\mu}$$

$$= \int f''(x+y) \, d\tilde{\nu} \, d\tilde{\mu}$$

$$= (\tilde{\mu} * \tilde{\nu})(f'')$$

Kullish & chaichain?

6j for $gl(1|1)$

e^

$$\cancel{\Lambda^2(\Lambda^3 V) = \Lambda^6 V + S^2 V \otimes \Lambda^4 V}$$

$$\frac{n(n-1)(n-2)\,[\,n(n-1)(n-2)-6\,]}{72}$$

$$= \frac{n(n-1)(n-2)(n^3-3n^2+6n-6)}{72}$$

$$= \frac{n(n-1)(n-2)(n-3)(n^2+2)}{72}$$

$$V_m \otimes V_n \otimes R^1_n \otimes R^2_m \longrightarrow$$ rep of S_{m+n}

S_{m+n-1}

Witten, Feb 22 2000

start in two dimensions

w/ a "scalar field" ϕ

$$\phi : \underset{\substack{\uparrow \\ \text{Riemann surface.}}}{\Sigma} \longrightarrow S^1 = \mathbb{R}/2\pi\mathbb{Z}$$

$$\mathcal{L} = \frac{R^2}{4\pi} \int_\Sigma |d\phi|^2$$

R = "radius of the circle"

$$Z = \frac{\text{Numerator}}{\sqrt{\det{}' \nabla}} = \frac{2\pi R \sum_x \exp\left(-\frac{R^2}{4\pi} |x|^2\right)}{\sqrt{\det{}'\Delta}}$$

$\prod$ nonzero eigenvalues

$x \in H^1(\Sigma, \mathbb{Z})$

$$|x|^2 = \int \partial\varphi \, \bar\partial \varphi$$

almost unique harmonic φ.

$$\phi = \phi_+ + \phi_-$$

Harmonic. $\quad 0 = \bar\partial \phi_+ = \partial \phi_-$

Decomposition not quite unique, up to a constant

Is there a Q.TM Theory of Φ_+ only?

$d\phi_+$ = self-dual 1-form.

$R=1$ a simple ϕ

Equivalent to free fermions:

$$\mathcal{L}_\psi = \int d^2x \left(\psi \bar\partial \psi + \tilde\psi \partial \tilde\psi \right) \qquad \begin{array}{l} \psi \in \Gamma(S_+) \\ \tilde\psi \in \Gamma(S_-) \end{array}$$

classical eqns: $\bar\partial \psi = 0 \quad \psi \leftrightarrow \phi_+$

$\partial \tilde\psi = 0 \quad \tilde\psi \leftrightarrow \phi_-$

$\bar\psi \psi \leftrightarrow \partial \phi_+ \ldots$

The theory of ϕ_+ only $\Rightarrow$ the theory of $\psi, \bar\psi$

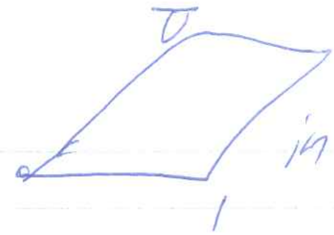
$\phi_- \longrightarrow \tilde\psi, \bar{\tilde\psi}$

Needs a spin structure

$Z_\alpha(\psi) = \det(\bar\partial_\alpha)$

$Z_\alpha(\tilde\psi) = \det(\partial_\alpha)$

$$Z(\phi) = \sum_\alpha Z_\alpha(\psi) Z_\alpha(\tilde\psi)$$

Concretely: Σ of genus 1 

in $\tau > 0$

$$Z_\alpha(\Psi) = \frac{\Theta_\alpha(\tau)}{\eta(\tau)} \qquad \eta(\tau) = \text{Dedekind } \eta$$
$$= q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$
$$q = e^{2\pi i \tau}$$

$\Theta_\alpha(\tau)$ = a theta fn of $H^1(\Sigma, \mathbb{R}) / H^1(\Sigma, \mathbb{Z}) = T$ $\left(\Theta\begin{bmatrix}\theta \\ \phi\end{bmatrix}(\tau) \quad \theta, \phi = 0, \tfrac{1}{2} \right)$

How do we build such a theta function:

1. need a cplx structure on T
   $(\alpha \to *\alpha)$

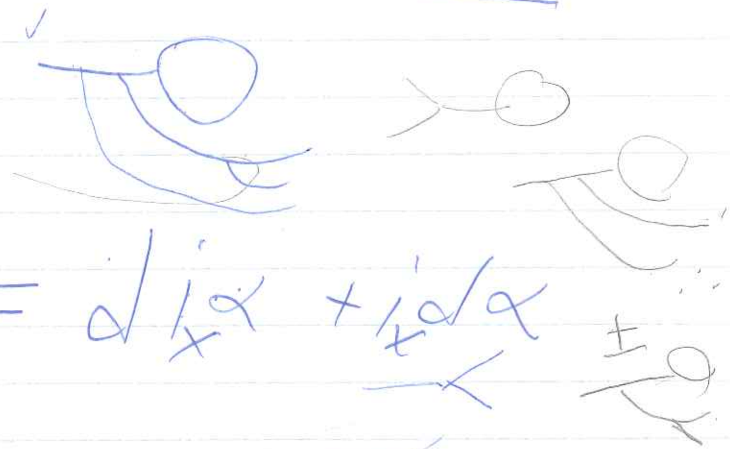
2. need a holomorphic line bundle $\mathcal{L}$ of degree 1

$\dim H^0(T, \mathcal{L}) = 1$

Θ = hol section of $\mathcal{L}$

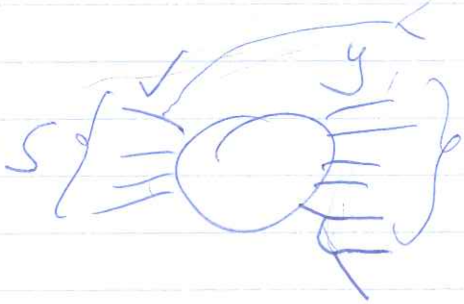
$c_1(\mathcal{L}) \in H^2(T, \mathbb{Z}) \leftrightarrow$ intersection pairing.

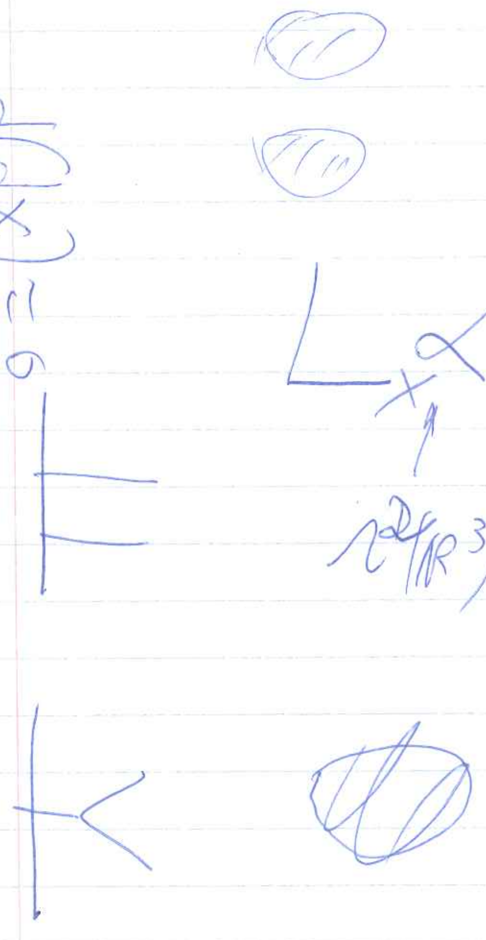
in fact, $\mathcal{L}$ has a connection with a constant curvature given by the two form $(a,b) = \int_\Sigma a \cup b$.



$$L_X \alpha = d\, i_X \alpha + i_X d\alpha$$

$\Omega^2(\mathbb{R}^3)$

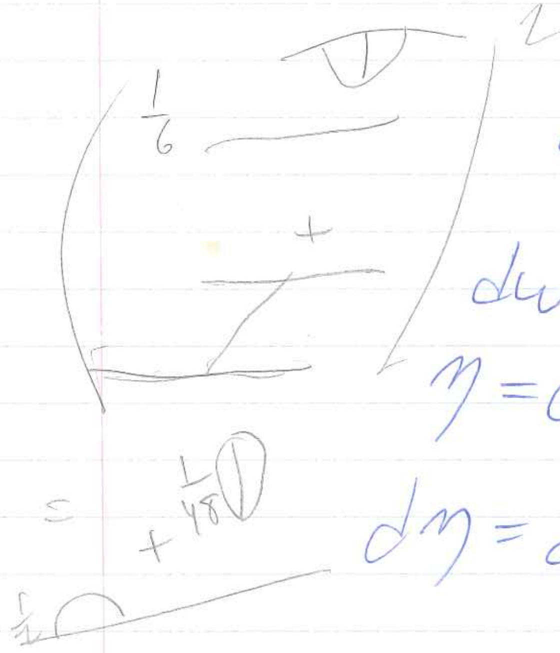




$$\theta_{L^2} = \{ d^*, L \}$$

$$d = \sum \theta\, dx_i \wedge \frac{\partial}{\partial x_i}$$

$C : \Omega^{n} \longrightarrow \Omega^{n-1}$



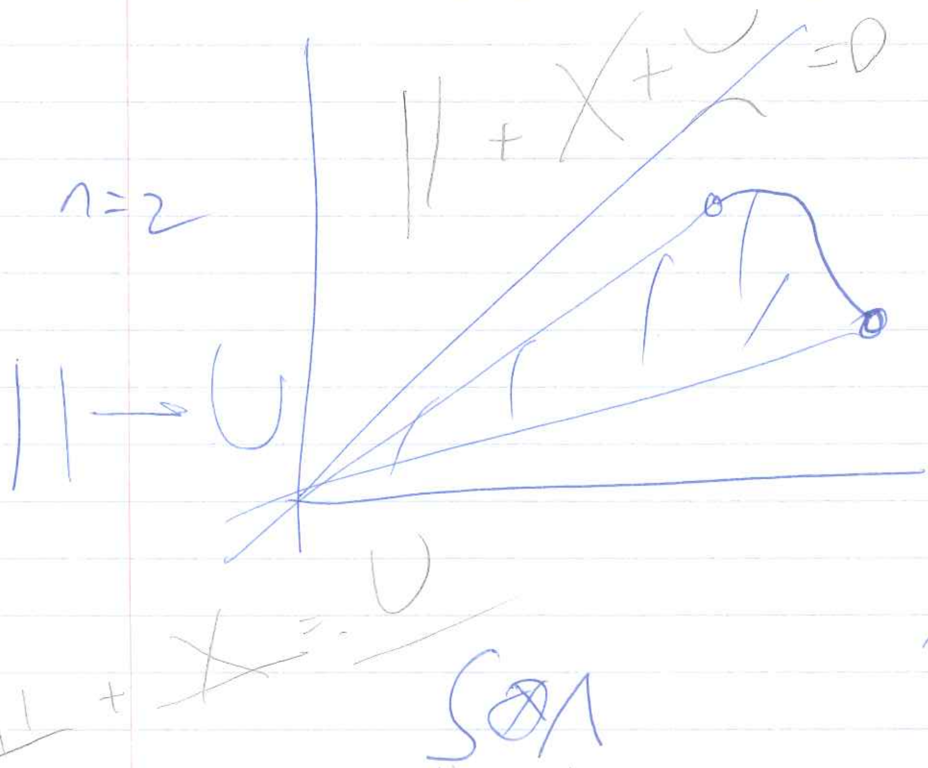
$dC + Cd = I$

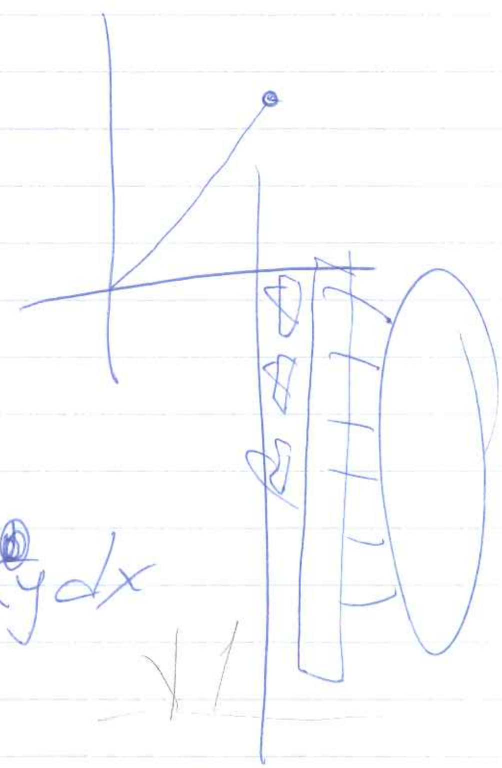
$d\omega = 0$

$\eta = C\omega$

$d\eta = dC\omega = Cd\omega \pm \omega \pm \omega$

$n = 2$



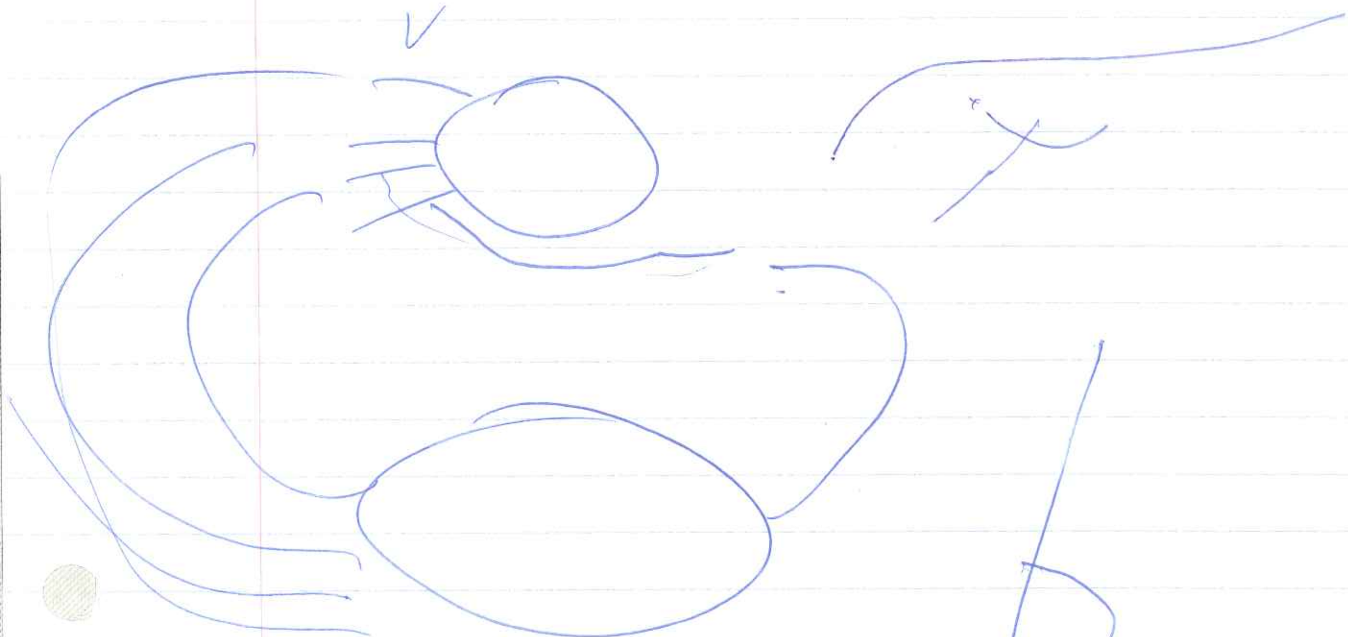


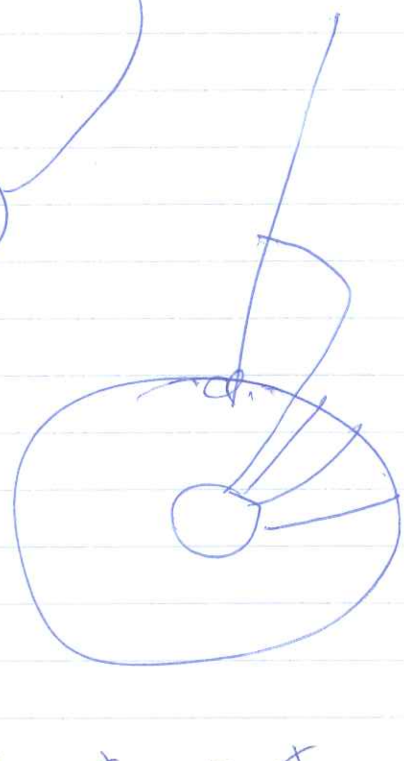
$\int \oslash \wedge$

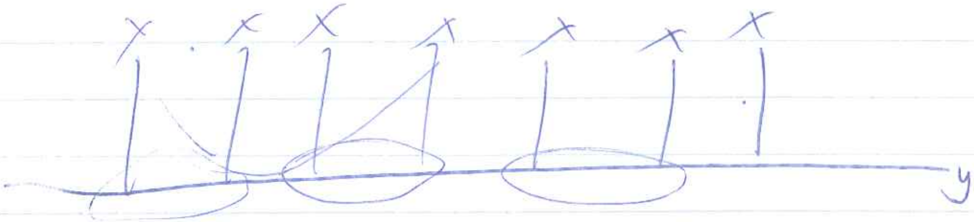
$\frac{1}{d}$

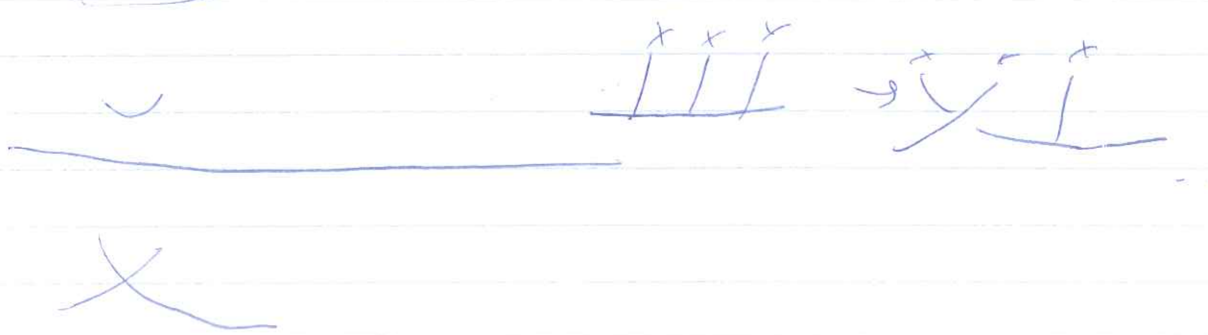
$x\,y\,dx$

$x_1 x_2 x_3$






y



$$\log Z(M,K) = -\frac{1}{2}\log \underline{\underline{\Delta(K)}}(e^{x}) + \underline{Q(M,K)}(x)\,\Theta$$

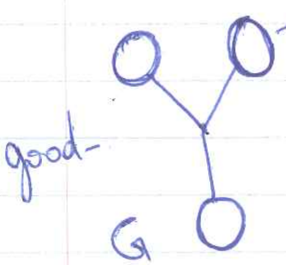
$Q(M,K) =$ hairy θ $=$  $\in \left(\dfrac{\mathbb{Q}[[x_1,x_2,x_3]]}{(x_1+x_2+x_3)}\right)^{Aut(\theta)} \in \hat{\Lambda}_\theta$

$\uparrow\ t_i = e^{x_i}$

$Aut(\theta)$: $Sym_3 \times \mathbb{Z}/2$

<u>Conj</u> $\boxed{Q(M,K) \in \Lambda_\theta} = \left(\dfrac{\mathbb{Q}(t_1,t_2,t_3)}{(t_1t_2t_3-1)}\right)^{Aut(\theta)}$

<u>Prove</u> $\boxed{\text{Do moves on } K}$

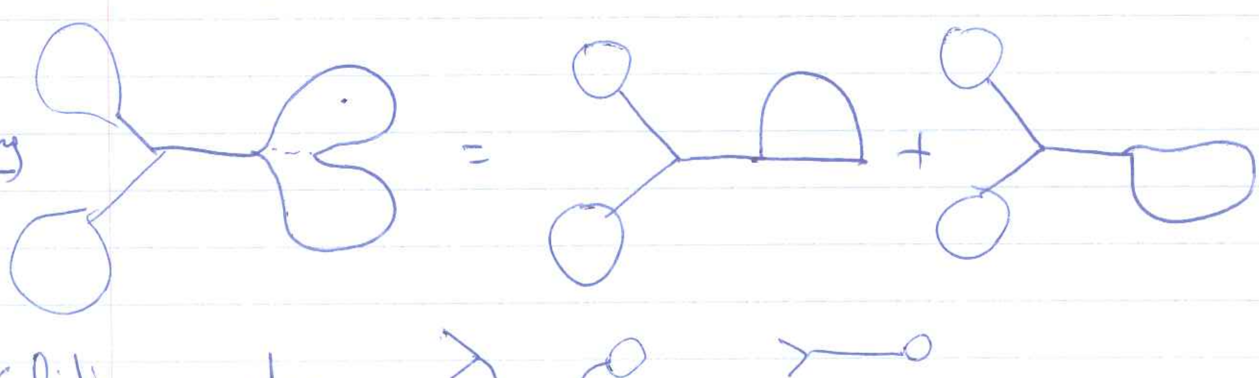


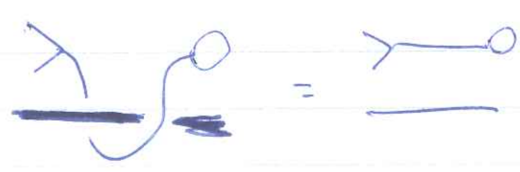
$G \hookrightarrow M-K$

$\boxed{(M,K)} \rightsquigarrow (M_G, K)$

$\exists\ \mathcal{F}^{loop}_\bullet$ filtration

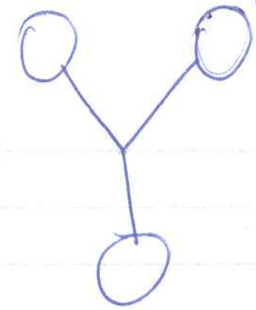
$\underline{\mathbb{Q}}\ \mathcal{G}^{loop}_n$

Cutting a leaf 

Sliding an edge 

$\boxed{[\Pi,\Pi]}$ $\Pi = \pi_1(M-K)$

<u>Problem</u>



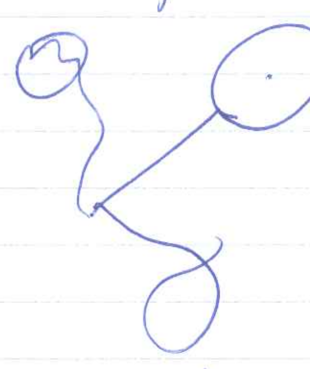
$\subseteq M-K$

If we knew good

that <u>leaves</u> $\in \underline{[[\Pi,\Pi],[\Pi,\Pi]]}$

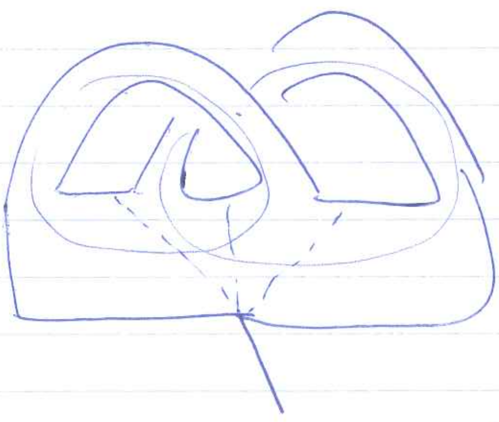
then

good graph



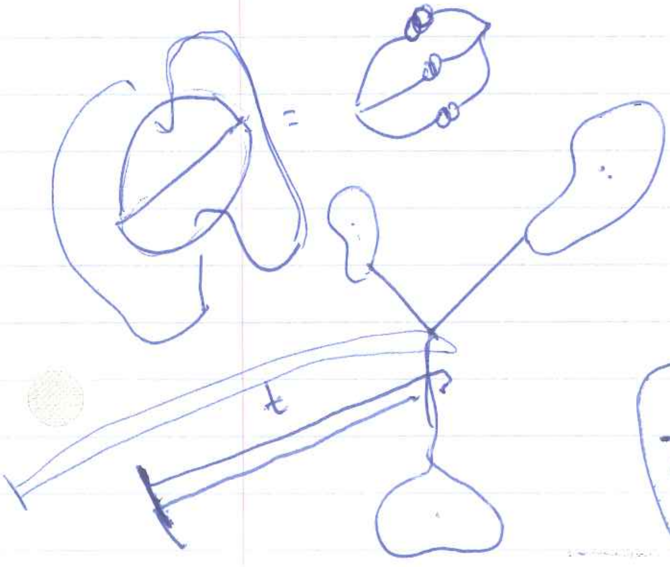
$\subseteq M-K$

then cut



$\boxed{\text{ETG}}$

with homology decorations



<u>Thm</u>: n-loop part of $\log Z$ is of ~~finite~~ type 2n
loop-finite type 2n

$\mathbb{Z}[t,t^{-1}]$: assuming RC
$\log Z$ is universal inv.
but in degrees >0

~~Thm~~

1-loop part of $\log Z$

$$= Q$$

having 

$$\begin{cases} Q(K_1 \# K_2) = Q(K_1) + Q(K_2) \\ Q(K^{\text{mirror}}) = -Q(K) \end{cases}$$

$$= \boxed{\text{type 2}}$$

$$Q[(M,K),G_1,G_2]$$

$$= \det\left(lk_{\tilde{M}}(L_{1i}, L_{2j}) \right)$$

$\boxed{\text{Kauffman: On knots}}$

"Kontsevich"
Z determines σ

Z does not

$\boxed{\sigma_\omega : S^1 \to \mathbb{Z}}$

cont. except at finitely many points

If A = Seifert matrix for K

$$\sigma_\omega(K) = \operatorname{sign}\left(\underbrace{(1-\omega)\cdot A + (1-\bar{\omega})A^T} \right)$$

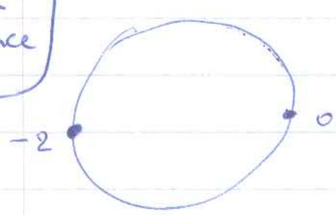
$$\boxed{\sigma_{-1}(K) = \operatorname{sign}(A + A^T)}$$

If Normalize at jumps

then

$\sigma : S^1 \to \mathbb{Z}$ is concordance invariant

When eigenvalue crosses at ω then $\Delta(K)(\omega) = 0$

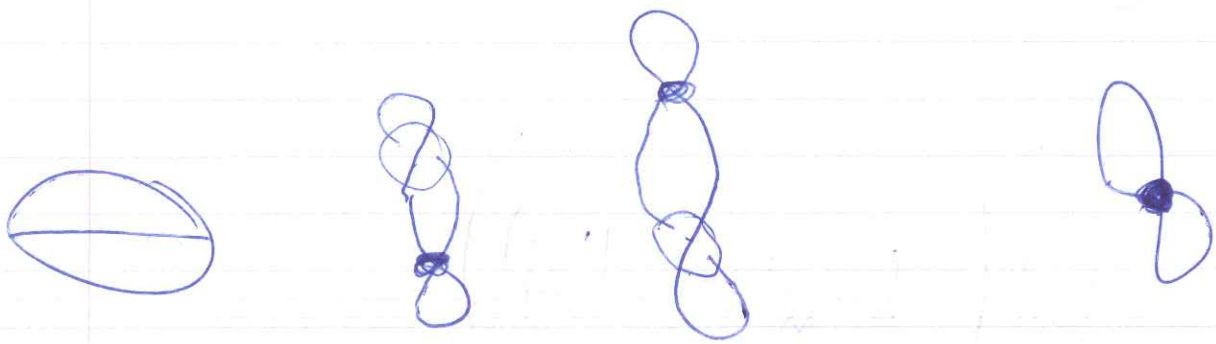


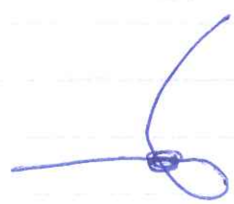
$$\sigma\begin{pmatrix}\text{right}\\ \text{trefoil}\end{pmatrix} = -2$$

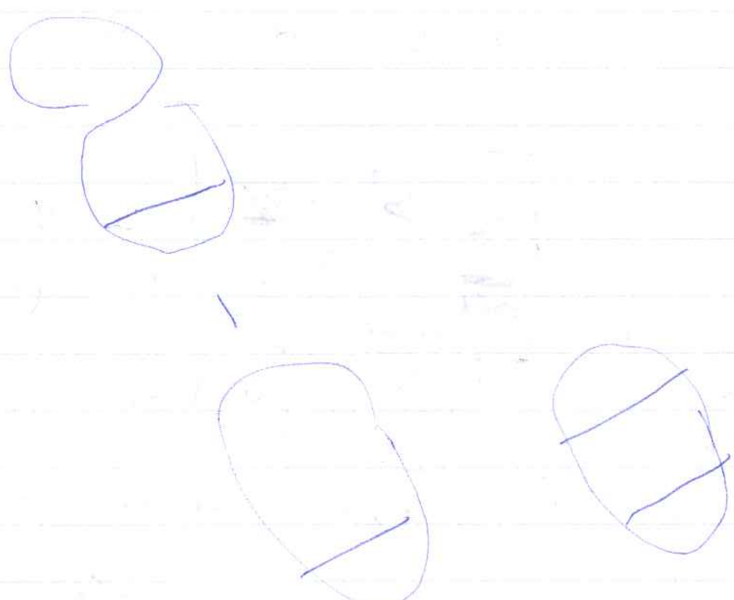
$$\Delta(T_{2,3})(t) = \frac{(t^{pq}-1)(t-1)}{(t^p-1)(t^q-1)}$$

$\Delta(K)(1) = 1$

$\Delta(K)(-1) = \text{odd} \neq 0$







The Q-function = having (?)

$\sigma(K_1 \# K_2) = \sigma(K_1) + \sigma(K_2)$

$\sigma(K^{mirror}) = -\sigma(K)$

$\sigma(K^{reverse}) = \sigma(K)$ $\neq Q(-1,-1,1)$

$\boxed{\text{behaves the same way}}$

$$\sigma_{-1}(K_+) - \sigma_{-1}(K_-) = \begin{cases} 0 & >0 \\ & \neq 0 \\ 2 & \Delta_{K_+}(-1)\Delta_{K_-}(-1) < 0 \end{cases}$$

$$\Delta_K(-1) = \det(A + A^T) = \pm \left| H_1\left(\begin{smallmatrix}\text{2-fold} \\ \text{branched} \\ \text{cover}\end{smallmatrix}\right)\right|$$

$$= \pm J_K(-1)$$

$\cancel{\Delta_{K_+}(-1) - \Delta_{K_-}(-1) = 2i}$

The separation theorem:
(Kostant, 68)

ad g submodule

$$\mathcal{U}(\mathfrak{g}) = H \otimes Z(\mathfrak{g})$$

Kostant even shows (but Bernstein-Luntz don't (1986~))

that H can be taken to

be

$$(\operatorname{ad} \mathcal{U}(\mathfrak{g}))(\mathcal{U}(\mathfrak{n}^-))$$

Annihilation thm for classical Lie alg

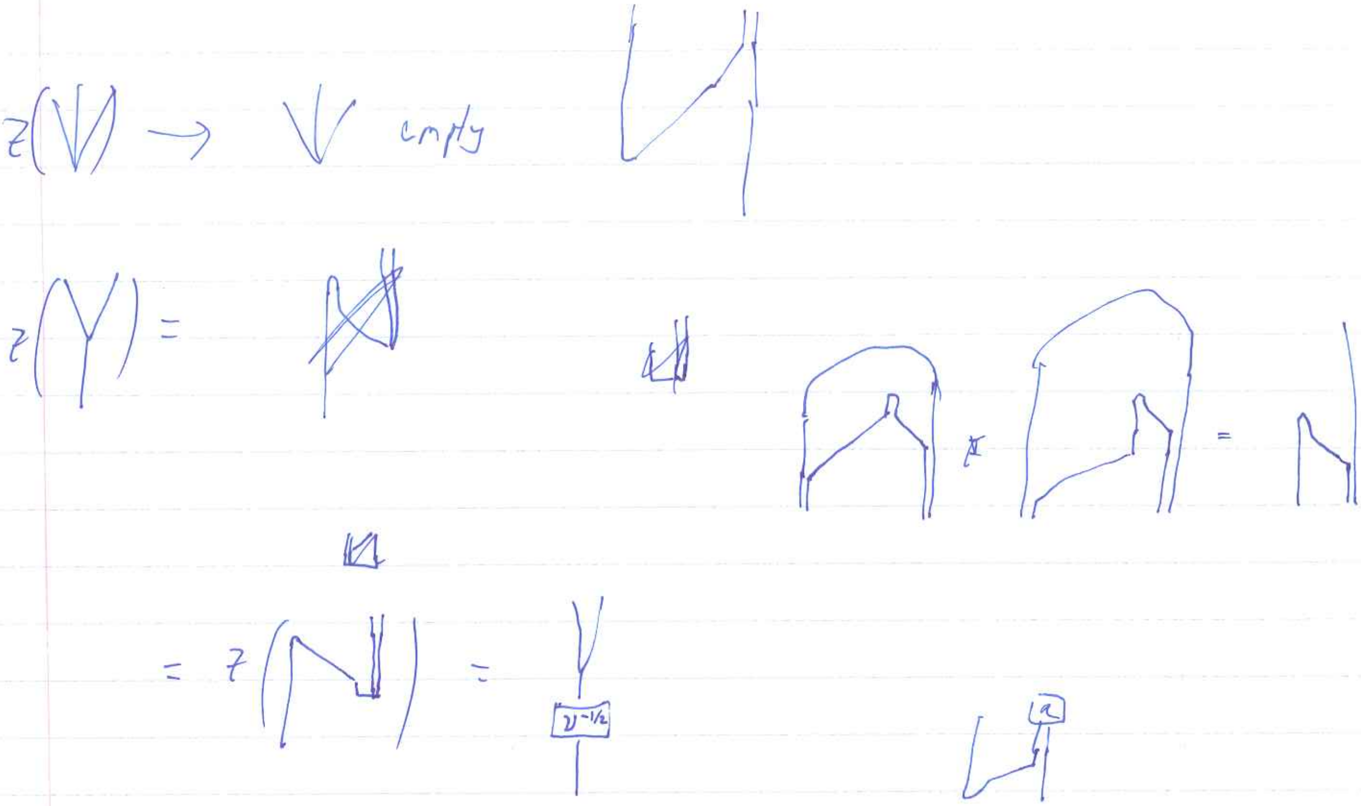
Any minimal ~~prime~~ primitive ideal is centrally generated and is the annihilator of a Verma module. $\Longleftrightarrow \operatorname{Ann}_H M = 0$

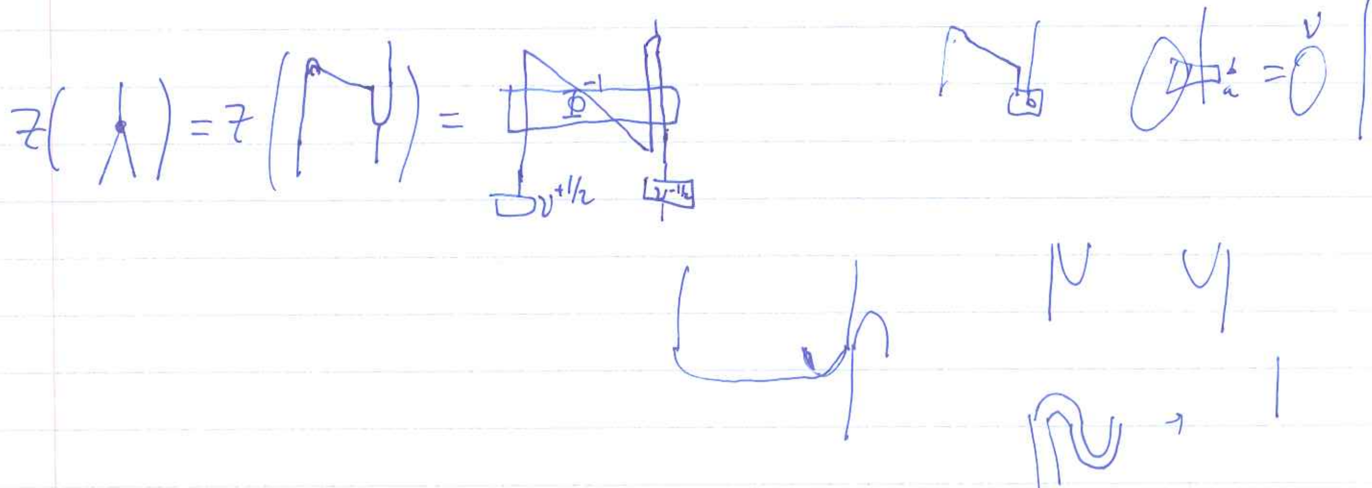
Verma mod.

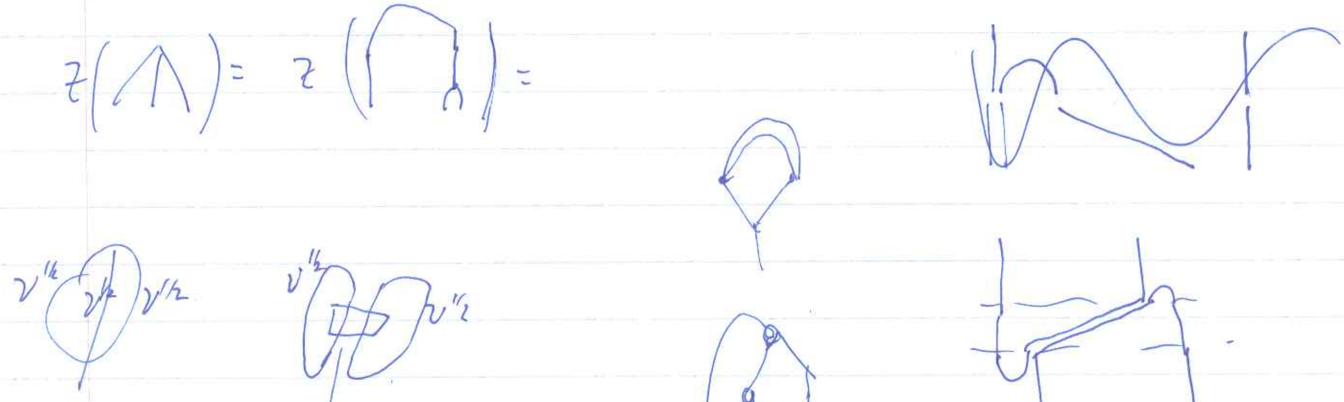
originally by Duflo 1971
new proof by Joseph & Letzter

Good source: Joseph in Proc of Montreal summer School, 1997, or his book.

Book on superlie algebra (1979) Scheunert, LNM 716
Also hep-th/9607161



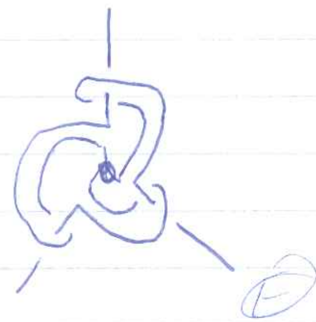




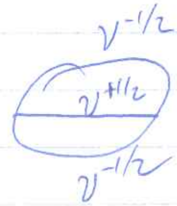
$\Delta(\nu^{1/2})$

correction:
$\nu^{1/2}\otimes\nu^{1/2}\otimes\nu^{-1/2}$

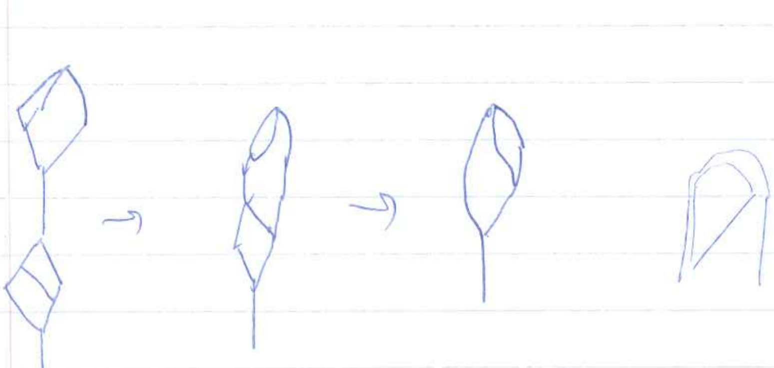
$\Delta(\nu) = \nu\otimes\nu$ mod link

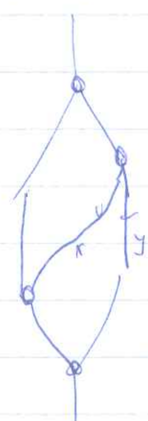


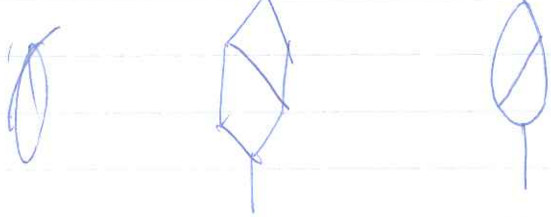
μ:



$f(x,y) + f((-x-y), x+y)$
$- f(-x-y, x) - f(-y, y)$

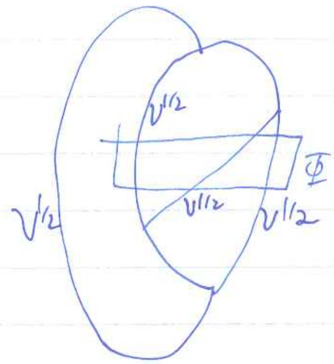


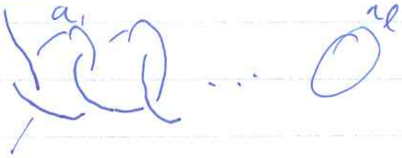




$f(x,y) + f(-x-y, x+y) - f$

The ~~te~~ tetrahedron:



$S := S_{x_i x_i}(a_i) :=$ 

$$\prod \hat\Omega^{-1}_{x_i} Z(S) = \left(\hat\Omega^{-1}_{x_1} Z\left(\uparrow^{x_1}_{1_x}\right)\right)\cdot \Omega^{-1}_{x_1}\cdot\left(\prod_{i=2}^{l-1}\Omega^{-1}_{x_i}\right)$$

$$\cdot\, e^{-\frac{\theta\sum a_i}{48}}\, e^{\sum\frac{a_i}{2} x_i x_i}\cdot\prod_{i=1}^{l-1} e^{x_i \frown x_{i+1}}$$

$$= \left[\hat\Omega^{-1}_{x_1}\left(\Omega_{x_1}\cup \exp_\# \xrightarrow{1^{x_1}} {}_x\right)\right]\cdot\prod_{i=1}^{l-1}\Omega^{-1}_{x_i}\, e^{x_i \frown x_{i+1}}\cdot e^{-\frac{\theta\sum a_i}{48}}\, e^{\sum\frac{a_i}{2} x_i x_i}$$

$$\xrightarrow{\sigma_x} \left[\hat\Omega^{-1}_{x_1}\left(\Omega_{x_1}\cdot e^{x \frown x_1}\right)\right]\prod_{i=1}^{l-1}\Omega^{-1}_{x_i}\, e^{x_i\frown x_{i+1}}\cdot e^{-\frac{\theta\sum a_i}{48}}\, e^{\sum\frac{a_i}{2} x_i x_i}$$

$$\cancel{\prod \hat\Omega^{-1}_{x_i}\check Z(S) = \langle\Omega,\Omega\rangle^{-l}}$$

$$\left(\prod_{i=1}^{l}\hat\Omega^{-1}_{x_i}\right)\left[\prod_{i=1}^{l}\nu_{x_i}\, Z(S)\right] = \langle\Omega,\Omega\rangle^{-l}\,\hat\Omega^{-1}_{x_1}\left(\Omega_{x_1}\, e_\#^{\xrightarrow{1^{x_1}}{}_x}\right)\cdot\Omega_{x_l}\cdot e^{-\frac{\theta\sum a_i}{48}}\cdot e^{\frac{1}{2}\sum a_{ij}\, x_i x_j}$$

TOC:

I Introduction: 1. A knotted trivalent graph is an up-to-isotopy..

2. The moves

3. Our main points:

a. under these moves, KTG is finitely generated

b. & finitely presented.

c. Thus invariants are "easily found" in ~~(U)~~ TG-sets.

Furthermore, we ~~show~~ explain why TG-set valued invariants of KTGs are interesting:

d. $U_g(\Gamma)$ and Drinfel'd associators.

e. 6j symbols

f. $A(\Gamma)$ and NAT & q-tangles

g. $M(\Gamma)$ and the asymptotics of CSW

At the same time, there are plenty things we do not yet understand:

1. the proper algebraic setting
2. Chern-Simons
3. Vazirani

II: The Murakami-Ohtsuki invariant.

III Finite generation

IV Finite presentation.

V our questions.

Map Legend

A RESEARCH PROJECT/PROPOSAL/PROGRAM ON KNOTTED TRIVALENT GRAPHS

DROR BAR-NATAN AND DYLAN P. THURSTON

ABSTRACT. TBW.

This pre-preprint is not yet available electronically at `http://www.ma.huji.ac.il/~drorbn`.

CONTENTS

1. INTRODUCTION

1.1. Acknowledgement. We wish to thank Greg Kuperberg and Justin Roberts for comments and suggestions.

REFERENCES

[B-N1] D. Bar-Natan, *On the Vassiliev knot invariants,* Topology **34** 423–472 (1995).

[B-N2] ——, *Bibliography of Vassiliev Invariants,* `http://www.ma.huji.ac.il/~drorbn/VasBib`.

ABOUT THIS PAPER...

1.1. Revision History.

June 5, 2000: Writing began.

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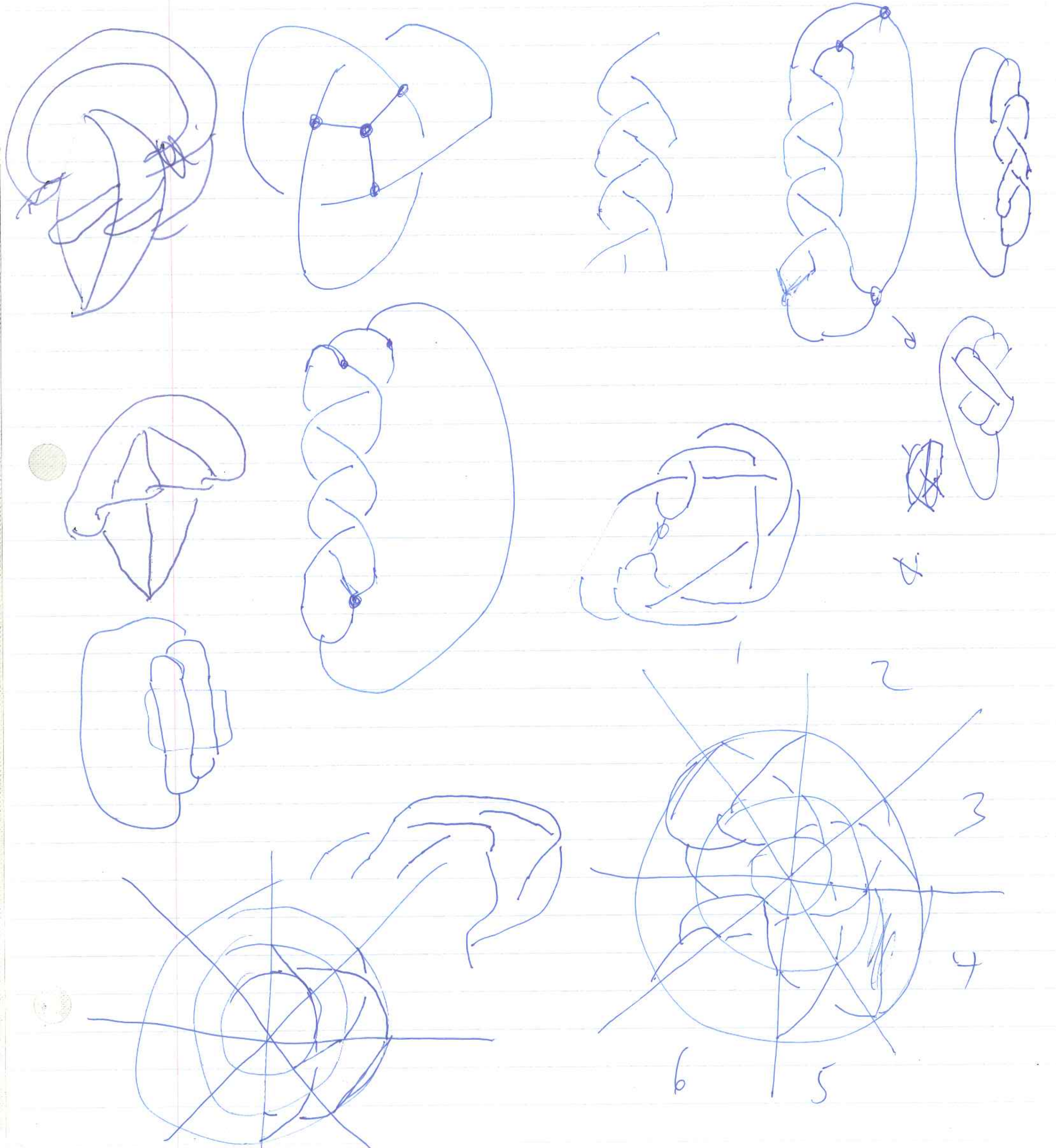
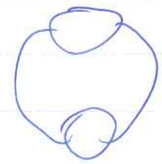
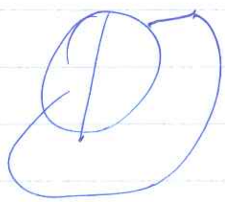
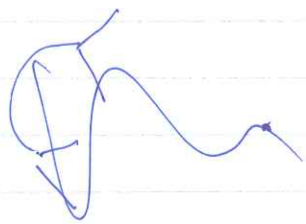
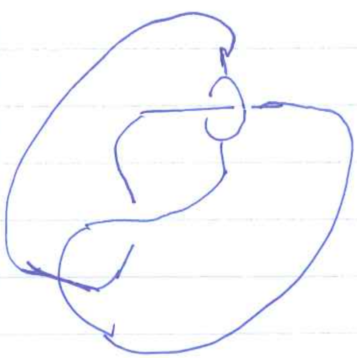
Date: This Edition: Jul. 20, 2000; First Edition: **This is merely an unifinished draft**; Revision History: Section 1.1.

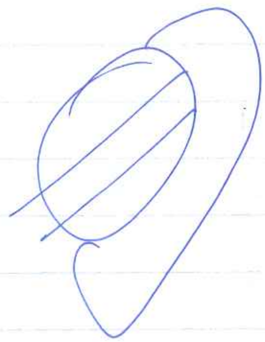
$\mathring{A}_1$ ~~#~~ is ~~#~~multiplying $Z(L^\circ)$ by (per component)



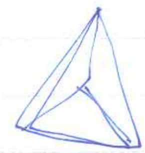


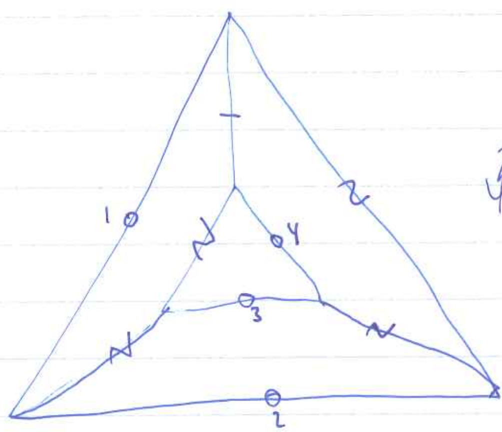







$$\Omega_{\alpha x}\Omega_{\beta x}\, e^{\frac{\gamma}{2} x \frown x}$$

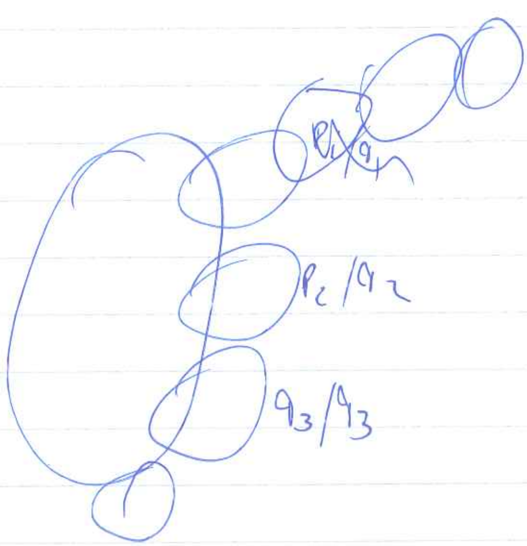


$$\Omega^{-1}\Omega_{\alpha x}\Omega_{\beta x}\Omega_{\gamma x}$$





$$\Omega\left(\Omega^{-1}\Omega_{x/\eta}\right)$$



$\cancel{\Omega^{-1(k+1)-2}}$ $\quad \Omega^{2-k}\,\Omega_{p_1}\Omega_{p_2}\Omega_{p_3}\dots e^{(\Sigma p_i)_x \frown x}$

$n=3$

p_i arb.

q_i det.

b det.

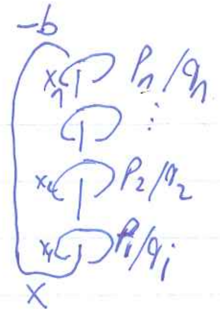
$\frac{2}{3} + \frac{12}{15} = \frac{22}{15}$

$2,3,5 \to$

$\frac{15\alpha+10\beta+6\gamma}{30}$ $\quad j \Leftrightarrow p=r=1$ $\quad \frac{1}{2}+\frac{1}{3}+\frac{1}{5}-1=\frac{1}{30}$

$1,2,4$

$$S^3(b;\ p_i/q_i) := \text{Surgery on}$$



$$=: L(b;\ p_i/q_i)$$

$$\Lambda = \begin{pmatrix} -b & 1 & 1 & 1 & 1 \\ 1 & f_1 & & & \\ 1 & & f_2 & & \\ 1 & & & f_3 & \\ 1 & & & & f_4 \end{pmatrix}$$

$$\det \Lambda = -b\prod f_i - \left(\prod f_i\right)\left(\frac{1}{f_1}+\dots+\frac{1}{f_n}\right)$$

$$= -e\prod f_i \quad \text{with } e = +b + \sum f_i^{-1}$$

$$|H_1| = \left(\prod p_i\right)\cdot e = \prod p_i\left(b+\sum \frac{q_i}{p_i}\right)$$

$$\begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_4 \end{pmatrix}$$

(invariant of x after surgery on $x_1 \dots x_n$)

$$Z^{/x} = \langle \Omega, \Omega^{-1}\rangle \exp\frac{\theta}{48}\left(b+\sum s\left(-\frac{q_i}{p_i}\right)+\frac{q_i}{p_i}\right)$$

$$\Omega^{1-n}\prod \Omega_{x/p_i}$$

$$\exp \frac{-b}{2}\widehat{x\ x} - \sum \frac{q_i}{2p_i}\widehat{x\ x}$$

$$= \langle \Omega, \Omega\rangle^{-1}\exp\frac{\theta}{48}\left(b+\sum\left(\frac{q_i}{p_i}-s\left(\frac{q_i}{p_i}\right)\right)\right)$$

$$\cdot\,\Omega^{1-n}\prod \Omega_{x/p_i}\quad \cdot \exp\frac{\widehat{x\ x}}{2}\left(-b-\sum\frac{q_i}{p_i}\right)$$

$$\check{Z}^{/x} = \overbrace{\langle \Omega, \Omega\rangle^{-2}\exp\frac{\theta}{48}\left(b+\sum\left(\frac{q_i}{p_i}-s\left(\frac{q_i}{p_i}\right)\right)\right)}^{c_1}$$

$$\cdot\,\Omega^{2-n}\prod \Omega_{x/p_i}\cdot \exp\frac{\widehat{x\ x}}{2}\left(-b-\sum\frac{q_i}{p_i}\right)$$

$$\hat{A} = C_1 \int \Omega^{2-n} \prod \Omega_{x/p_i}\, e^{\frac{x \frown x}{2}\left(-b-\sum q_i/p_i\right)}$$

$$= C_1 \left\langle \Omega^{2-n} \prod \Omega_{x/p_i},\ \exp \frac{x \frown x}{2e} \right\rangle$$

With

$$C_1 = \langle \Omega, \Omega \rangle^{-2}\, e^{\frac{\theta}{48}\left(e - \sum s(q_i/p_i)\right)}$$

Coeff of $\frac{\theta}{48}$: $\left(e - \sum s(q_i/p_i) + \boxed{\frac{2-n}{e}} + \frac{1}{e} \sum \frac{1}{p_i^2}\right)$

Orlik Seifert Manifolds. #291 Springer Lectures.

Waldhausen. 1967.
2 papers Groups with centre and 3-mflds. I, II.

Homeom = Hom Equiv = Fibre pres homeom
≥ 4 except fibres.

---

3 except fibres. P. Scott Annals 1983
There are no fake Seifert fibre spaces

$z_1^p + z_2^q + z_3^r = 0$.

Milnor Brieskorn homology spheres
pp 175–225 Ann. Math Studies #84
Fox memorial; Neuwirth ed.

Berkeley Lecture V & VI, May 9 2000

1. Reminder ETG, moves, $\mathcal{A}$, Z

Z is determined by $Z(\Delta)$ & $Z(\bigcirc^{1/2})$

$Z(\bigcirc^{1/2}) = e^{\frac{1}{4}}_{\#}$

<u>Goal</u>: $Z'_{\phi} = Z^F \quad F \in \mathcal{A}(\ominus)$

2. NAT & rel. with ETG.

3. Pent & hex for NAT, relation w/ tensor categories

4. The Hochschild complex

5. $\mathcal{D}$

6. Homology.

Berkeley Lecture VII, May 11, 2000

- ☐ Reminder
- ☐ uniqueness & d^2
- ☐ non-obstruction & d^4
- ☐ Compute H
- ☐ Wrap up

$$\frac{q_1}{p_1} + \frac{q_2}{p_1} = \frac{1}{p_1 p_2} \qquad \frac{q_1}{p_1} = \frac{1}{p_1 p_2} - \frac{q_2}{p_1} = \frac{1 - q_2 p_2}{p_1 p_2}$$

$$q_1 p_1 + q_2 p_1 = 1$$

$$\cancel{\frac{p_1 p_2}{1 - q_2 p_2}}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \cancel{dd}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad ad - bc = 1$$

$$\frac{a}{c} - \frac{b}{d} = \frac{1}{cd}$$

$$\begin{matrix} 1 & 1 \\ 2 & 3 \end{matrix}$$

$$S\begin{pmatrix} a & b \\ c & d \end{pmatrix} := 3\sigma - \tau + \frac{a+d}{c}$$

$$S\begin{pmatrix} b & -a \\ d & -c \end{pmatrix} = 3\sigma' - \gamma + \frac{b-c}{d}$$

$$\frac{a+d}{c} - \frac{b-c}{d} = \frac{d(a+d) - c(b-c)}{cd} = \frac{1 + d^2 + c^2}{cd}$$

$$= \frac{1}{cd} + \frac{d}{c} + \frac{c}{d}$$

Jc

Curtis Moore

W. = The planar embedding weight system:

$$\langle e^{\frac{1}{2}\cup}, \Omega \rangle = e^{\theta/48}$$

$\eta(K_2\, K_4 \ldots) = \#$ planar connected 2-connected graphs w/ K_{2j} verts of valency j

$$H(X_2\, X_4 \ldots) = \sum_{K_2 K_4 \ldots} (2b_2 X_2)^{K_2} (b_4 X_4)^{K_4} \cdots h^{\sum j K_j} \cdot \eta(K_2\, K_4 \ldots)$$

Claim:

$$H(p, p^2, p^3 \quad) \cong \frac{\theta}{48}$$

Question: Does

$$H\left(2-n+\sum_1^n p_i,\ 2-n+\sum p_i^2, \ldots\right)$$

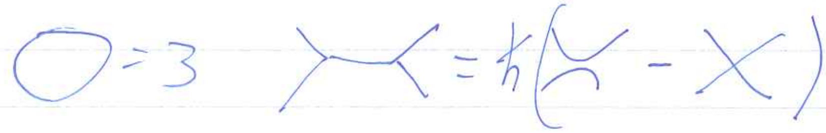
determine n & p_i's?

$\sum p_i^k$

(For $n \geq 3$)

$$e^{\frac{\hbar}{12}\sum \frac{1}{e p_i^2} + (2-n)} \cdot \left\langle e^{\frac{\vee}{2e}}, \left(\frac{\sinh\sqrt{\frown}/2}{\sqrt{\frown}/2}\right)^{2-n} \prod_{i=1}^{n} \frac{\sinh(\sqrt{\frown}/2p_i)}{\sqrt{\frown}/2p_i} \right\rangle$$

The A_1 relations

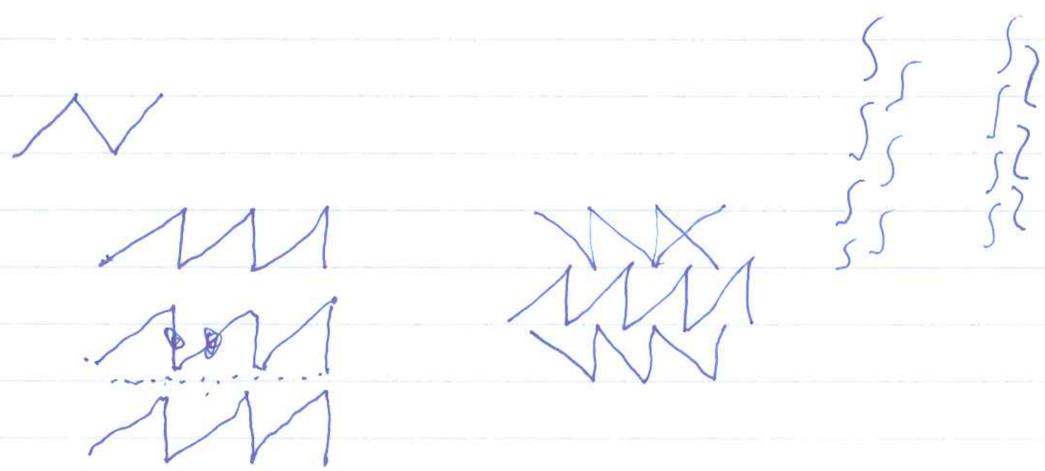


$\mathcal{A}^{sl(2)}(\emptyset)$ $\quad \mathcal{A}'(\emptyset) = \mathcal{A}(\emptyset) / A_1$-relations.

$$\simeq \mathbb{Q}[[\hbar]]$$

$$\mathcal{A}'(*) \simeq \mathbb{Q}[[s,\hbar]]$$

$$Z^{rast}(M) = \left\langle \frac{2\prod_i p_i \sinh\sqrt{\hbar s/4ep_i^2}}{\hbar s/2e \, \sinh\sqrt{\hbar s/4e}^{\,n-2}} \right\rangle e^{\frac{\hbar}{4e}\left(n-2-\sum \frac{1}{p_i^2}\right)}$$



f_{xx}, $f_{xy} = f_{yx}$, f_{yy} s.t.

$\partial_y f_{xx} = \partial_x f_{xy}$ $\quad$ $\partial_y f_{yx} = \partial_x f_{yy}$

$\partial_y f_x \stackrel{?}{=} \partial_x f_y$

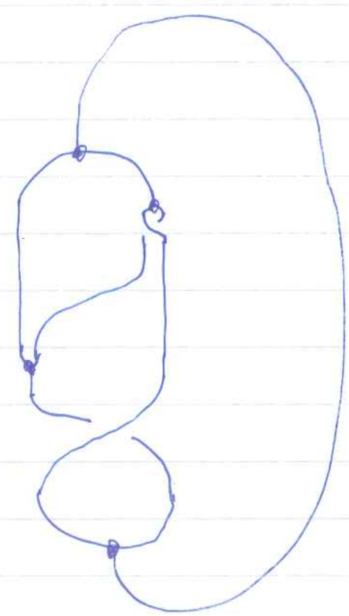
$f_{xy} = f_{yx}$

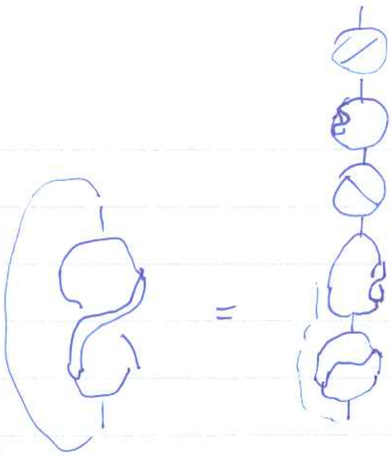
$s \in$ "$\frac{d}{d\theta}$"

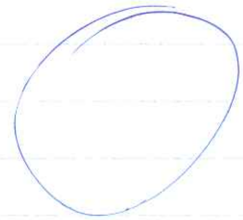
$$\hat{\theta} \stackrel{?}{\in} \mathrm{Coder}(\mathcal{A}) = \mathcal{U}_{\hat{\epsilon}}$$

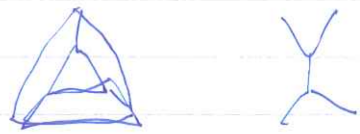
$$\begin{tikzcd} \mathcal{A} \arrow[r, "\square"] \arrow[d, "\hat{\theta}"'] & \mathcal{A} \otimes \mathcal{A} \arrow[d, "(1\otimes\theta)+(\theta\otimes 1)"] \\ \mathcal{A} \arrow[r, "\square"] & \mathcal{A} \otimes \mathcal{A} \end{tikzcd}$$

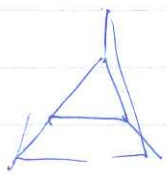
Dylan's $\boxtimes$:

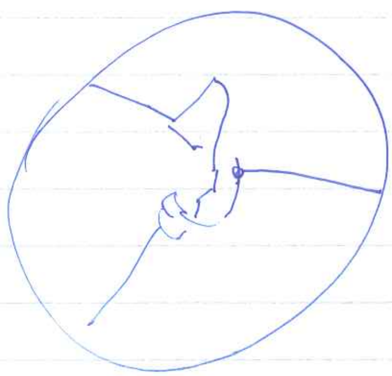


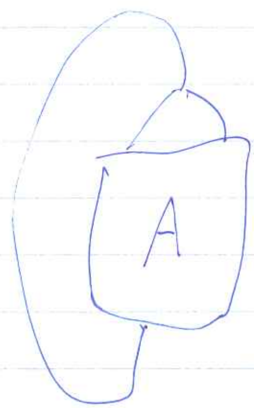








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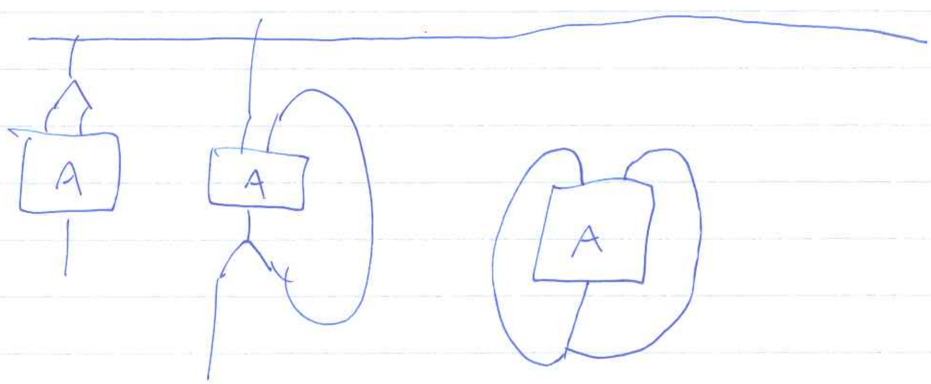
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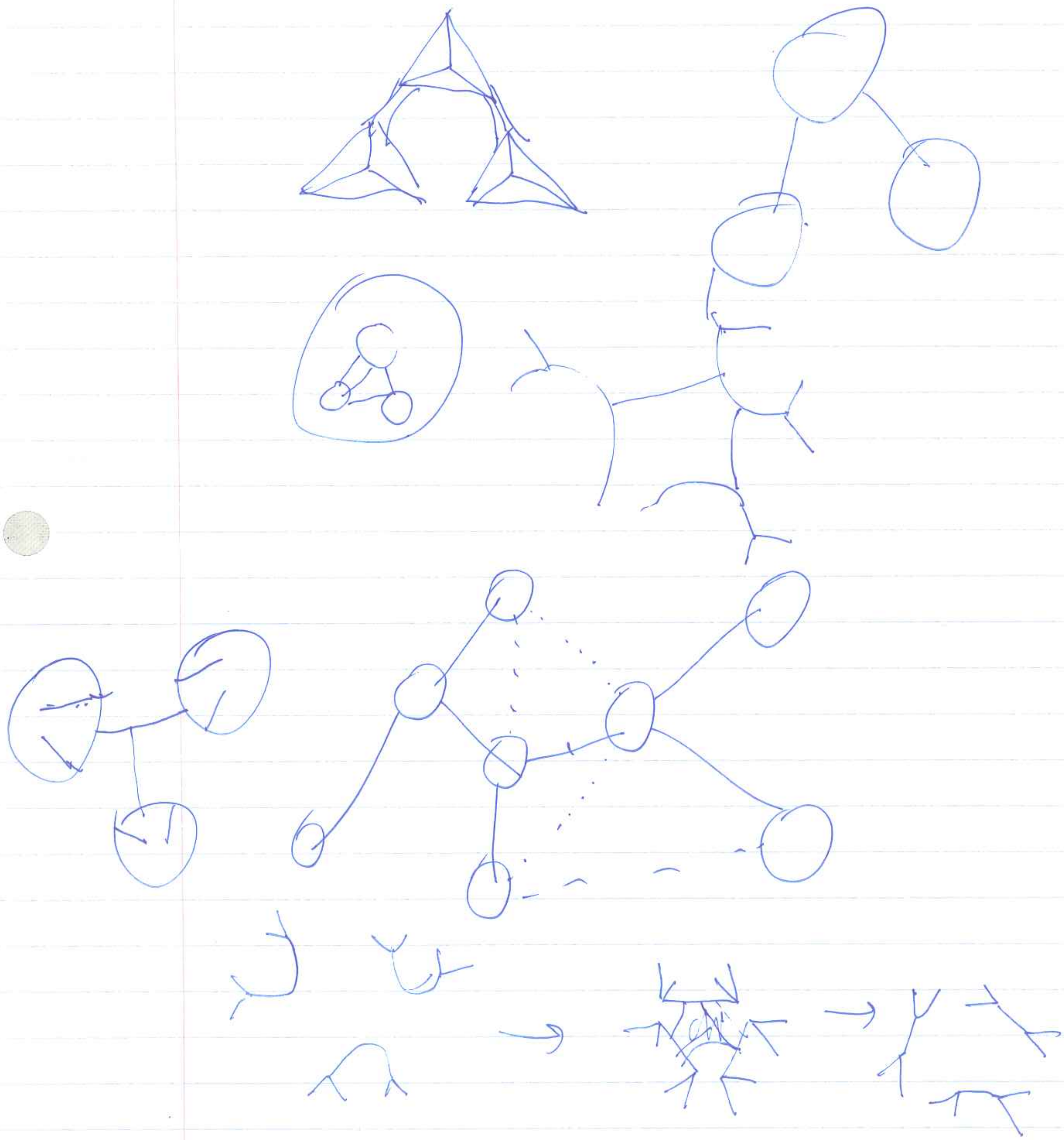
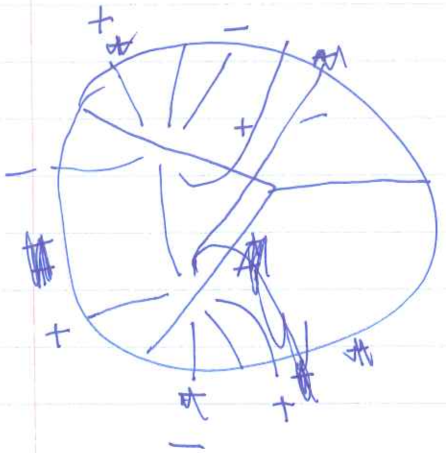
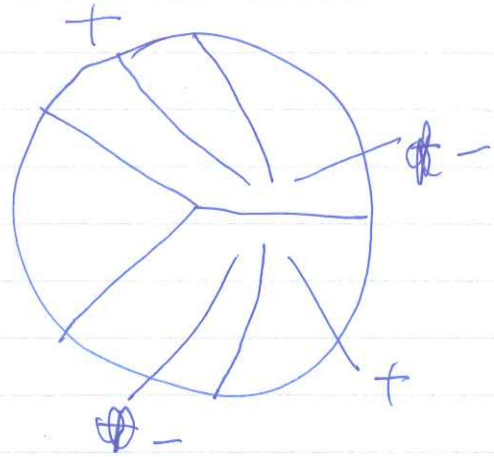
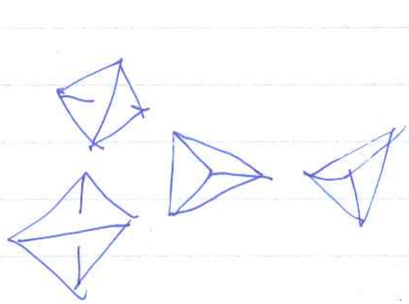
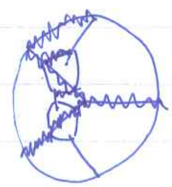
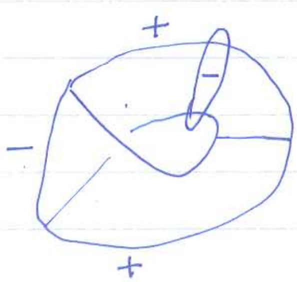
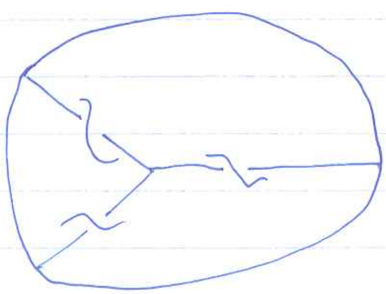
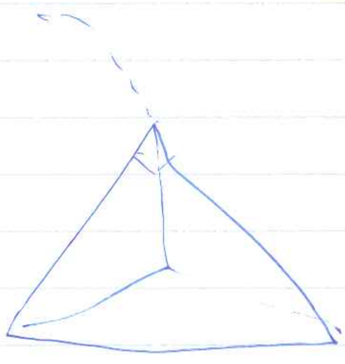
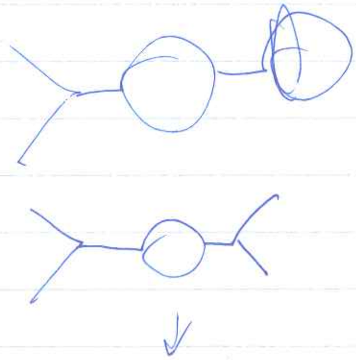
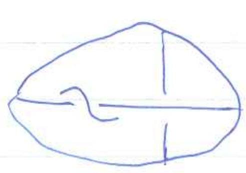
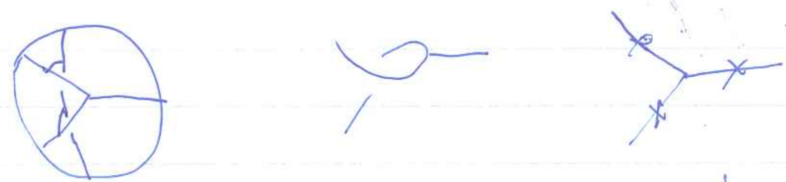
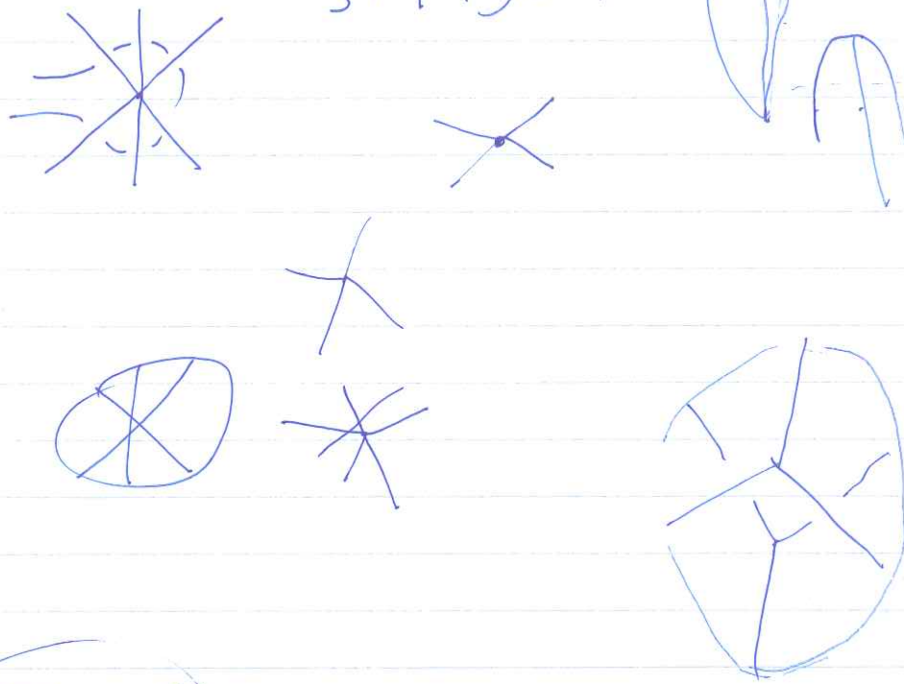
~~Rod Downey: Calibrating Randomness.~~

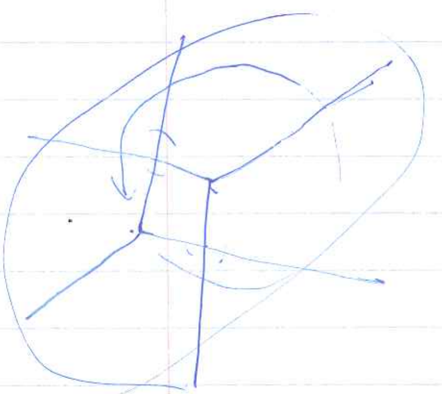


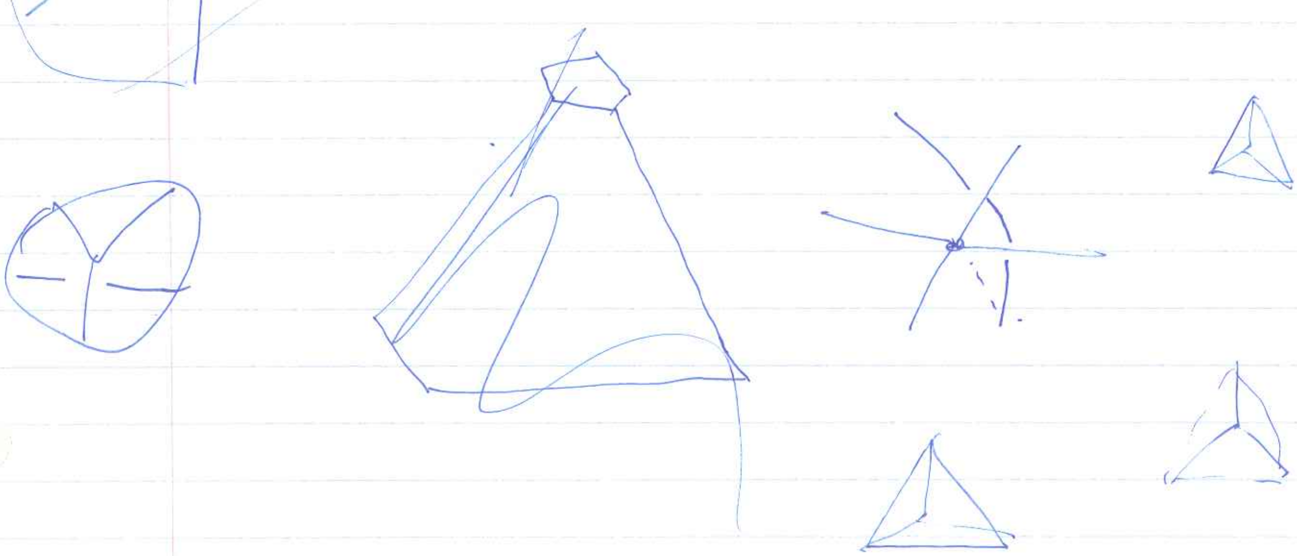



$3+1+3=7$





$1(1(2)3)4)5$

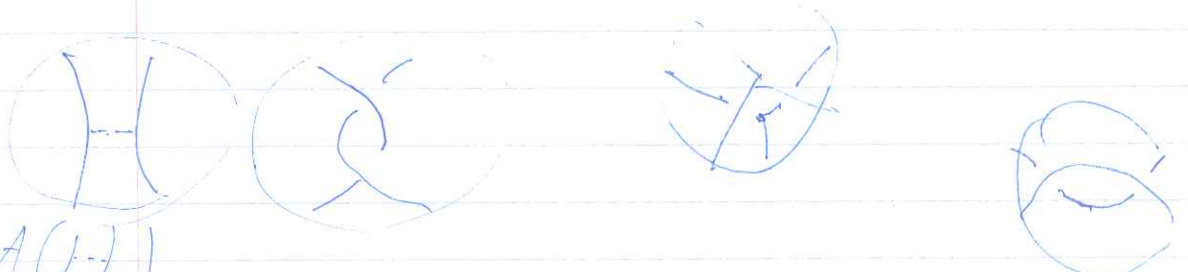


$$F \xrightarrow{\Psi} (\Psi F)(z) = \sum c_i F(L_i z)$$

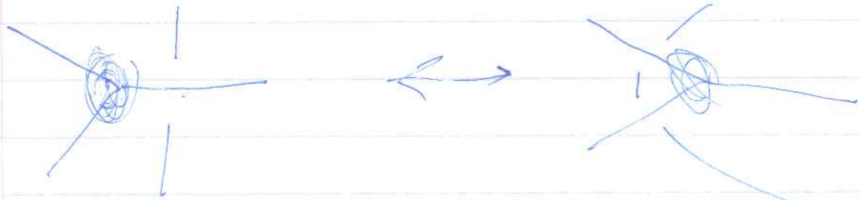
$\forall \Psi_1 \ \exists \Psi_2$ s.t.

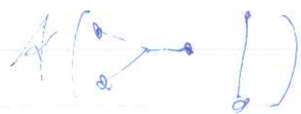
$$\mathrm{im}\,\Psi_1 \simeq \ker \Psi_2$$

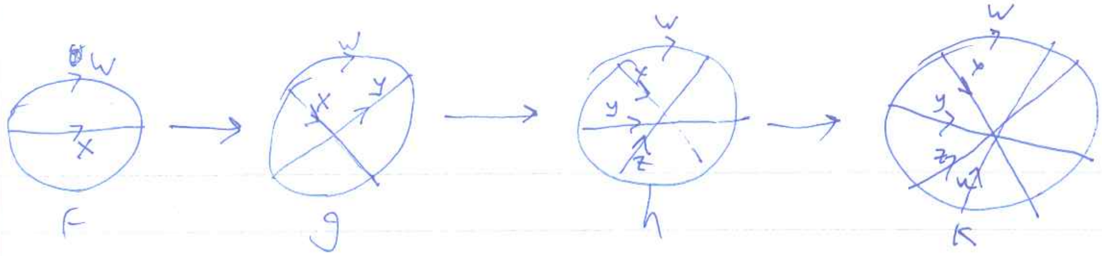
$$\begin{pmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{pmatrix}$$



A(|-|)

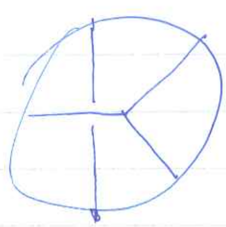


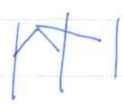




$$F(w,x) \longmapsto (dF)(w,x,y) = F(w+x,y) - F(w,y) + F(w+y,x) - F(w,x)$$

$$F(w+x+y,z) + F(w+x+z,y) + F(w+y+z,x)$$





$$\frac{(n+1)^{n+1}}{n^n} = \left(\frac{n+1}{n}\right)^n \cdot (n+1)$$
$$= (n+1)e$$

$$d\varphi(u,x) = \cancel{\varphi(u)}$$
$$\mu \qquad \varphi(u+x) - \varphi(u-x)$$
$$\frac{\partial}{\partial u}\mu(u,x) = \dot\varphi(u+x) - \dot\varphi(u-x) = 0$$
$$\varphi(u, \quad \Rightarrow \dot\varphi = const$$
$$\Rightarrow \varphi(u) = au^2 + bu + c$$

$$\mu(u,x) = a(u+x)^2 + b(u+x) - a(u-x)^2 - b(u-x)$$
$$= 2aux + bx + 2aux + bx$$
$$\boxed{= kux + \ell x}$$

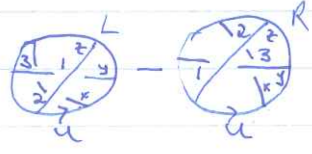
$$l_w \to l_w$$
$$l_x \to -l_z + l_w$$
$$l_y \to l_x$$
$$l_z \to l_y$$

$$\varphi(-u,-y,-x) = \boxed{\varphi(u,x,y) = \varphi(-u,y,x)}$$

$$\varphi(u,x,y) = \varphi(u+x+y, -x, -y) =$$

$$\varphi(-u,-x,y)$$
$$= (-1)^d \varphi(u,x,y)?$$

$$E_3(u,x,y,z)$$

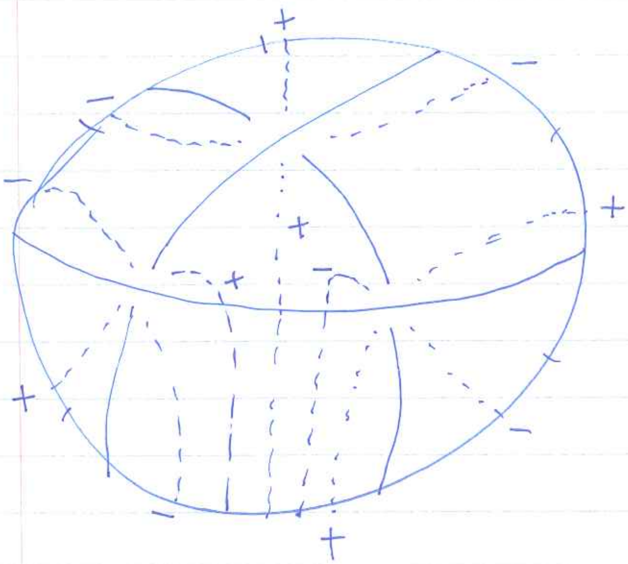


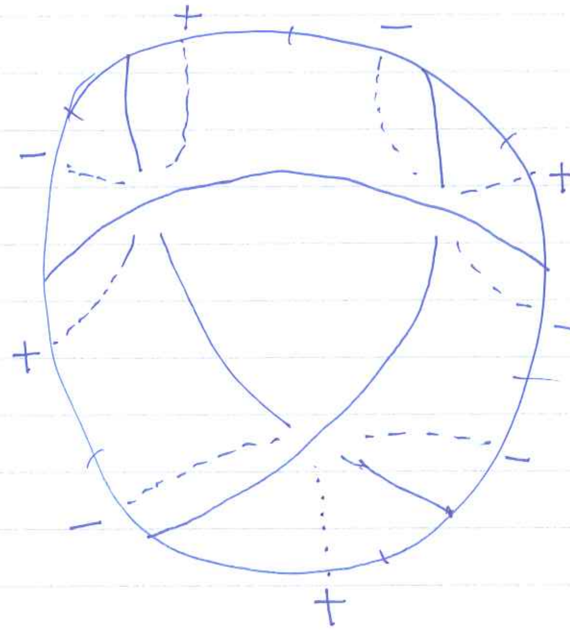
L R

$$\mu(u,x,y,z) = \varphi(u+x,y,z) - \varphi(u,y,z) \quad 1$$
$$+\varphi(u,x,z) - \varphi(u+y,x,z) \quad 2$$
$$+\varphi(u+z,x,y) - \varphi(u,x,y) \quad 3$$

$g(y,z) - g(x,z) + g(x,y) = 0$

claim $\quad g(x,y) = g(x,0) + g(y,0)$





~~ot~~ $F_k \mathcal{A}$ = things with at most k legs

$F_0 \mathcal{A} \subset F_1 \mathcal{A} \subset F_2 \mathcal{A} \subset \dots \subset F_k \mathcal{A} \subset F_{k+1} \mathcal{A} \dots$

$F_{k+1}\mathcal{A} / F_k \mathcal{A} \cong \mathcal{D}_{k+1}$ $\qquad 0 \to F_{k-1}\mathcal{A} \to F_k \mathcal{A} \to \mathcal{D}_k \to 0$

$0 \to$

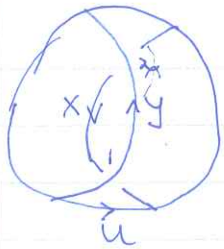
$\mathcal{A}_n = \mathcal{A}\left(\quad \right)$ n-1 chords

$\mathcal{A}_2 \xrightarrow{d} \mathcal{A}_3 \xrightarrow{d} \mathcal{A}_4 \xrightarrow{d} \mathcal{A}_5$

$7 \longrightarrow 7 \longrightarrow$

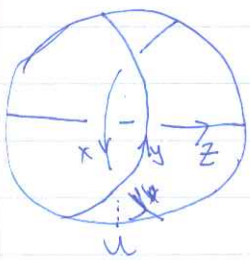
$7 \longrightarrow 6$

$\cancel{F(x+}$ $f(x,y) = f(x+y, y)$

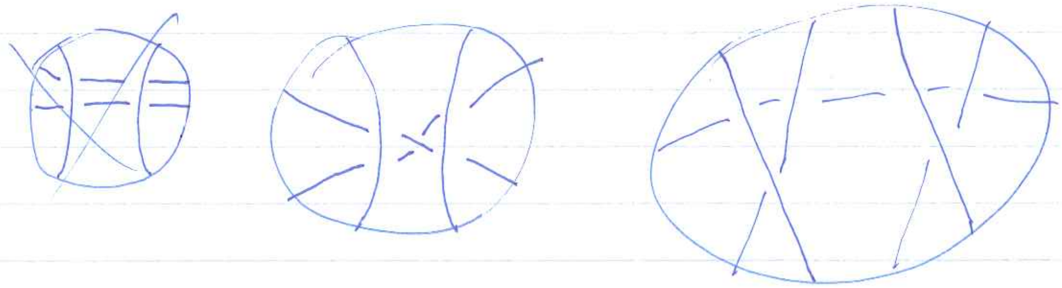


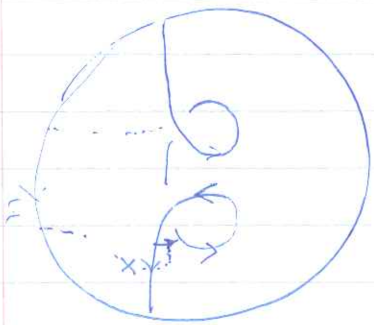
$$\psi(u,x,y) = \psi(u,x,y) + \psi(u+x, -x, y)$$

$$\psi(u+y, x, -y)$$



$\cancel{\psi(u,x,y) = A}$





$$\varphi(u,x,x) + \varphi(u+x, x, -x) = 0$$

$$\theta_1(x,y) - \theta_1(0,y)$$
$$- \theta_1(y,x) + \theta_1(0,x) = 0$$

$$\Delta\varphi = \theta(u+x, y) - \theta(u,y)$$
$$- \theta(u+y, x) + \theta(u,x)$$

$x\theta_1(y) - y\theta_1(x)$

$=$

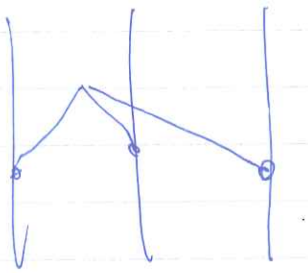
$\cancel{f_x(y) = f_y(x)}$ $\theta(a,b) = a^2 b$

$$\to y(x^2 + 2ux) - x(y^2 + 2uy)$$

$$yx^2 - xy^2$$

$\varphi(u,x,x) + \varphi(u+x, x, -x) = 0$

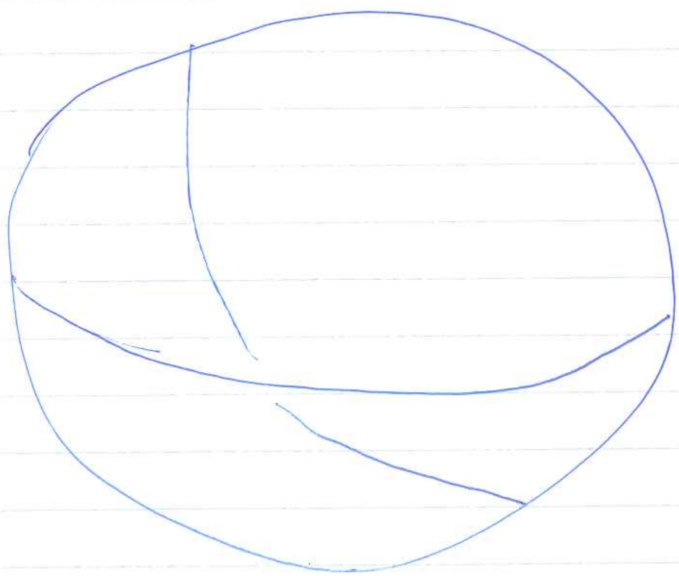
$\cancel{(x+y)^2 = x^2+y^2}$



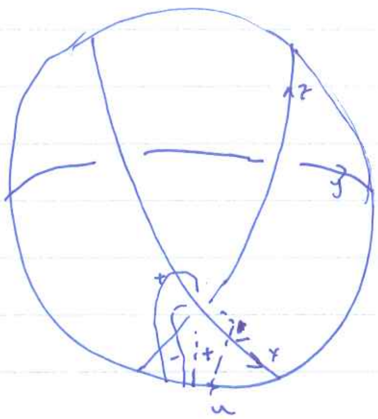
$$xyz \to yzw - (x+y)zw + x(y+z)w - xy(z+w) + xyz$$

$$d(xy^2) = yz^2 - (x+y)z^2 + x(y+z)^2 - xy^2$$

$2xyz$



$$(u+x+y)^2-(u+x)^2+(u+x+z)^2-\cancel{(u+x)^2}+u^2-(u+z)^2$$
$$+(u+y+z)^2-(u+y)^2-(u+x+y+z)^2+\cancel{(u+x)^2}$$

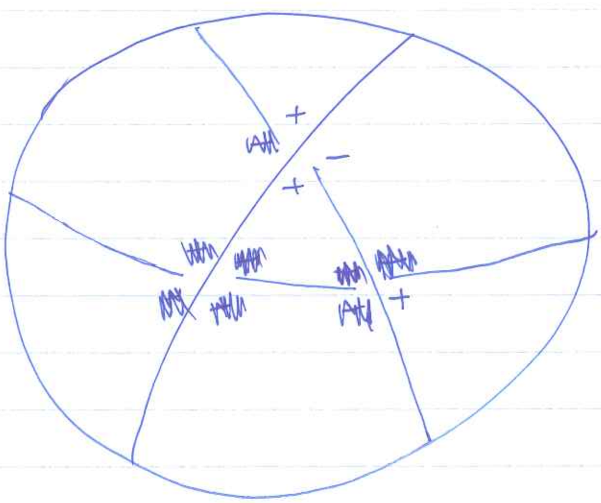


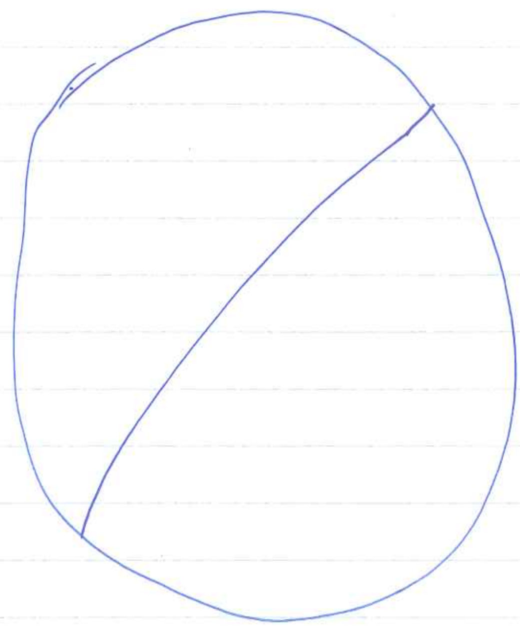
u^2: ✓ ux ✓ xy ✓

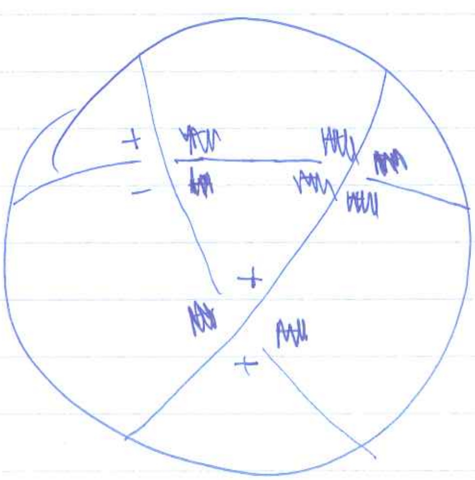
x^2: ✓ uy ✓ xz ✓

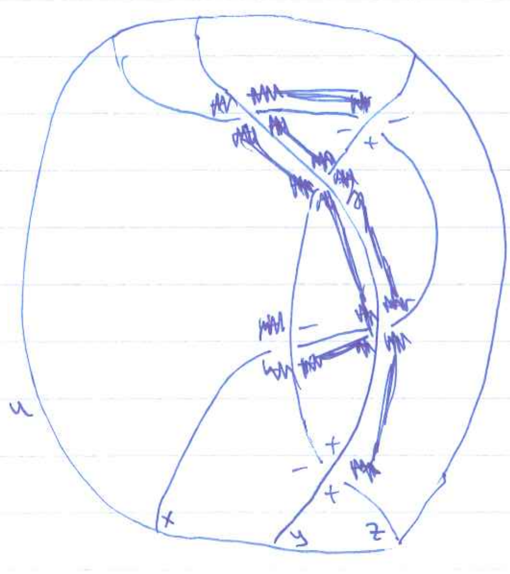
y^2: ✓ yz ✓

z^2 ✓



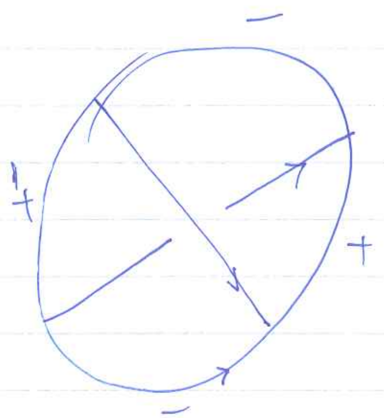






$$w^2 f(x) \longrightarrow (w+x)^2 f(y) - w^2 f(y) - (w+y)^2 f(x) + w^2 f(x)$$

$$= x^2 f(y) + 2wx f(y) - y^2 f(x) - 2wy f(x)$$



$$f(u,z) \rightarrow f(u+x,y) - f(u,y) - f(u+y,x) + f(u,x) = \cancel{g(x,y,u)}\ g(u,\cancel{x}y)$$

$$g(0,x,y) = f(x,y) - f(y,x) + f(0,x) - f(0,y)$$

$$e^{xy}$$

$$x^2 - y^2$$

$$w^2 \longrightarrow x^2 - y^2 + \underbrace{2w(x-y)}$$

$$wx \longrightarrow (w+x)y - wy - (w+y)x + wx =$$

$$= 0$$

$$wy \rightarrow 0$$

xy $\quad x^2 \rightarrow 2xy + y^2$

$\cancel{xy \rightarrow}$

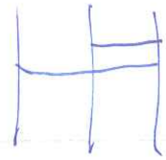
$\cancel{g(x+z,y)-g(}$

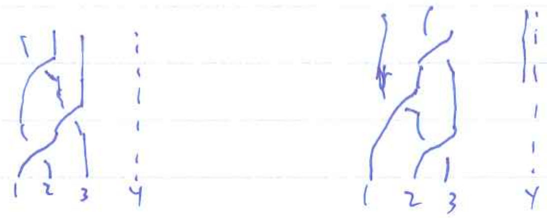
$$\left.\begin{matrix} xy \\ x^2 \\ y^2 \end{matrix}\right. \longrightarrow 0$$

$$g(x,y) = g(0,x+y) - g(0,x)$$

$$f(x+y) - f(0) - (f(x) - f(0))$$

$$g(x,y) = f(x+y) - f(x)$$

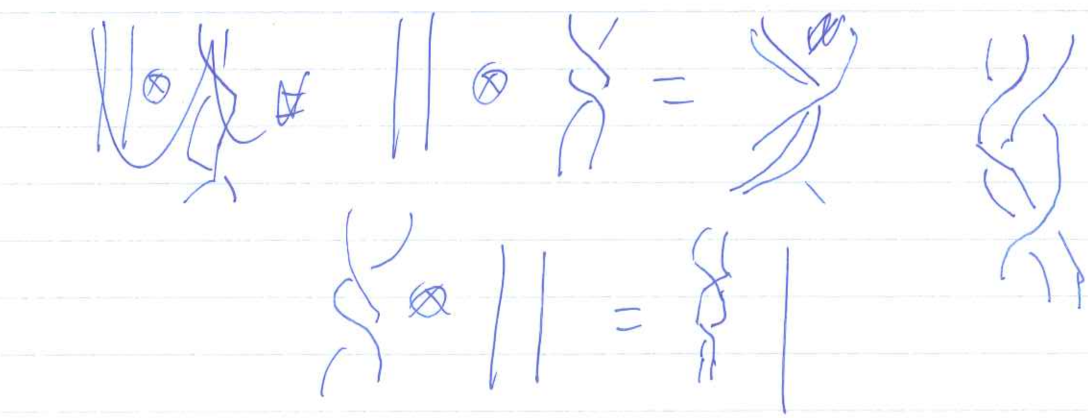




$$R^{1,2,34} \cdot R^{1,3,4} \cdot R^{2,3,14} = R^{2,3,4} \cdot R^{1,3,24} \cdot R^{1,2,4}$$

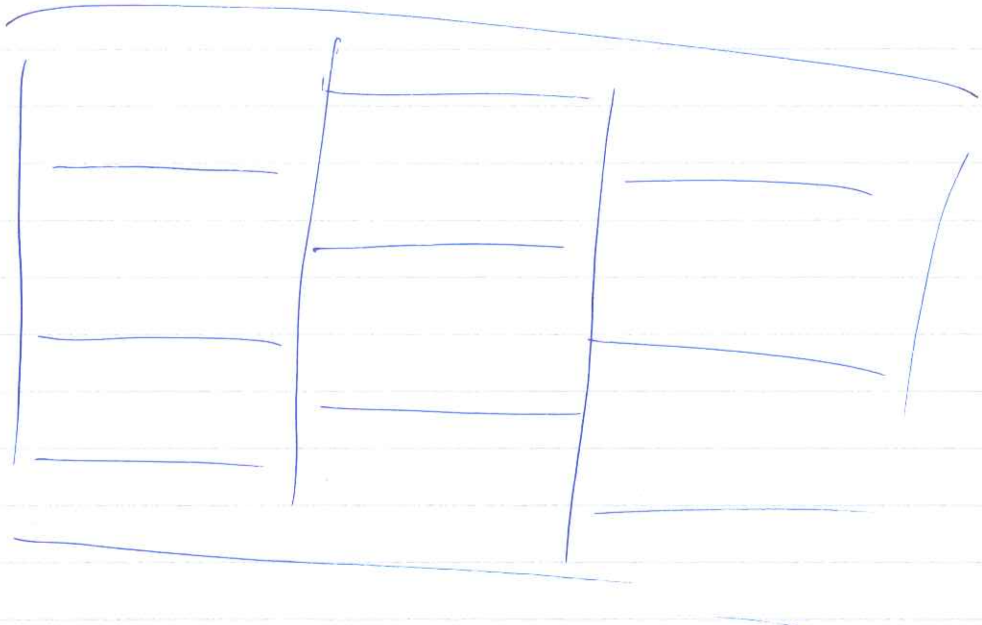
$$\rho(2,3,14) - \rho(2,3,4) - \rho(1,3,24) + \rho(1,3,4) + \rho(1,2,34) - \rho(1,2,4) = 0$$

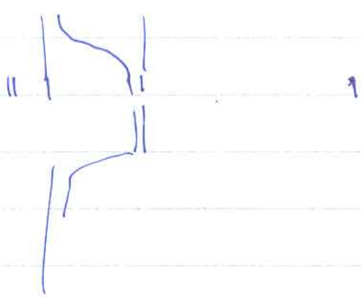
$A \otimes B :=$ 

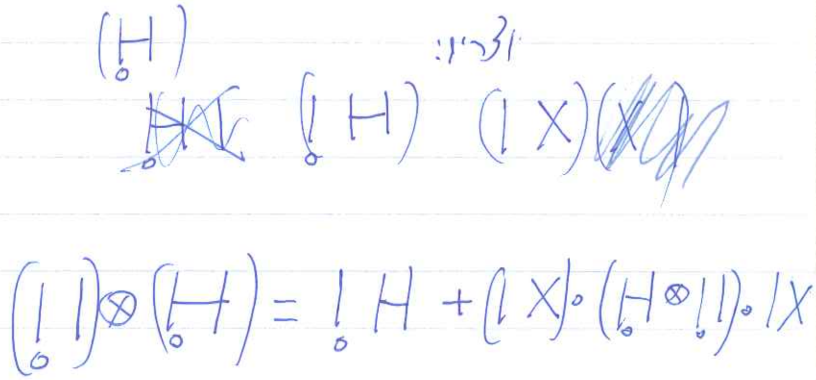


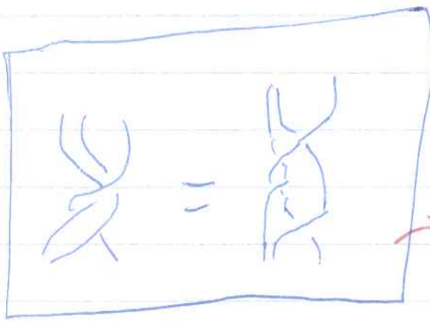
sourceforge.net
gimp-print.~~[illegible]~~

epson under linux.

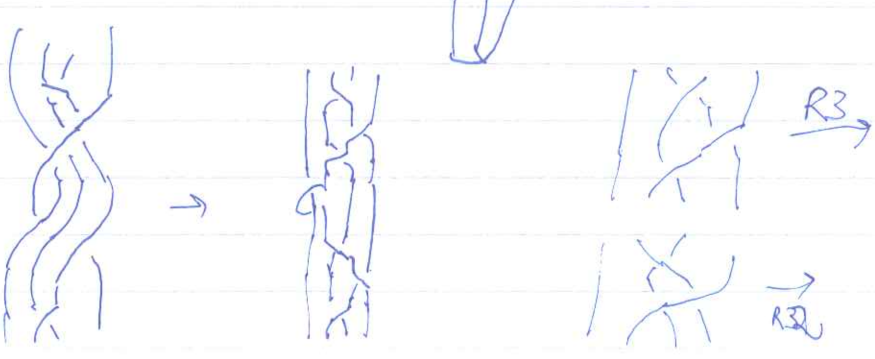




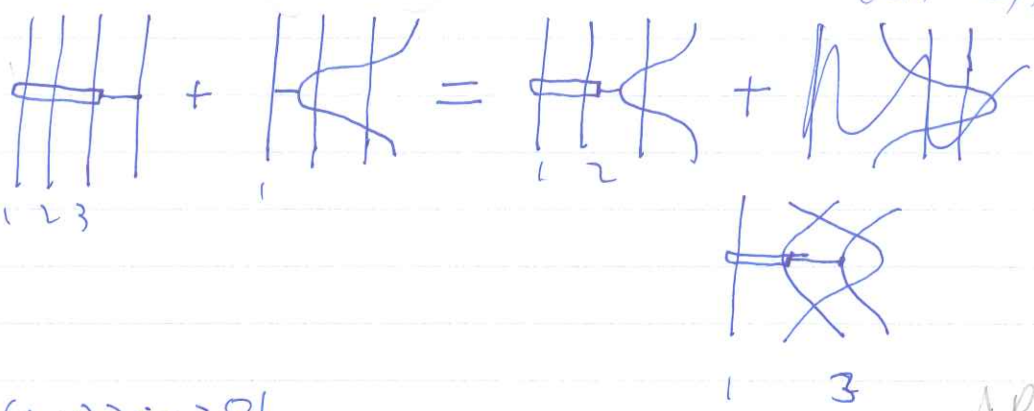




שרה נ"פרים
סכנות אורית מצוחית
6253004



Haering-Oldenburg
tom Dieck



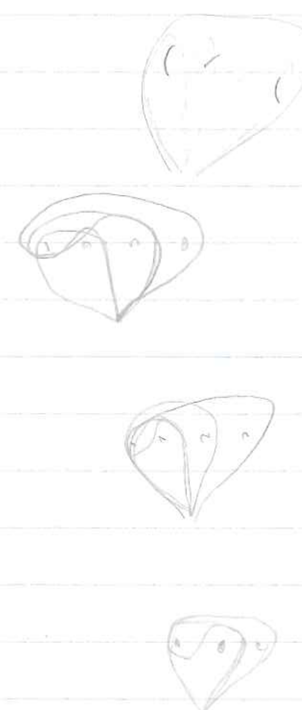
$(\Delta\otimes 1)\xi = \xi^{01}$

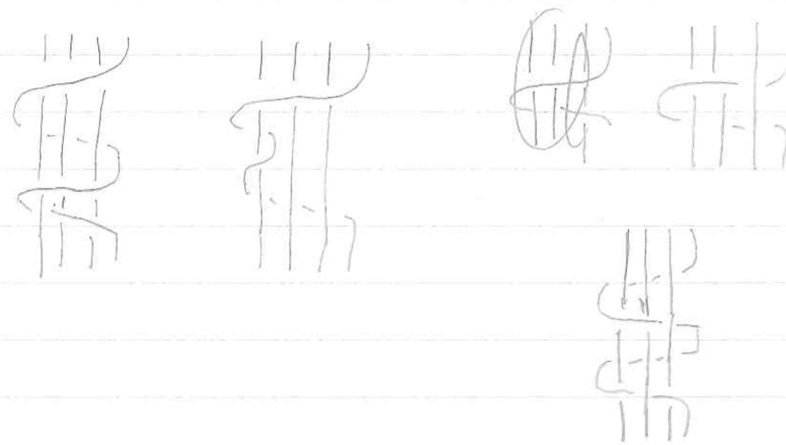
$(\Delta\otimes 1\otimes 1)\lambda = \lambda^{013} + \lambda^{023}$

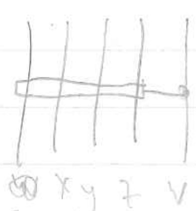
AB $\Rightarrow$ ACD

$\xi^{012,3} - \xi^{01,3} - \xi^{02,3}$

$- \xi^{01,2} + \xi^{0,1} + \xi^{0,2}$







$\psi(u+x, y, z, w) + \psi(\cancel{u, x, y}, w) + \psi(u, x, z, w)$

$\psi(u+y, x, z, w) + \psi(\cancel{u, y, x}, w) + \psi(u, y, z, w)$

$\psi(u+z, x, y, w) + \psi(u, z, x, w) + \psi(u, z, y, w)$

$$\cancel{[F^{01,23}, t^{03}]} + [F^{0,1,3}, t^{03}] - \cancel{[F^{01,2,3}, t^{03}+t^{13}]}$$

$$- \cancel{[F, t^{03}+t^{13}]} - [F^{02,1,3}, t^{03}+t^{23}]$$

$$= -[F^{01,2,3}, t^{13}] + [F, t^{13}] + [F^{0,1,3}, t^{03}]$$

$$- [F^{02,1,3}, t^{03}+t^{23}]$$

$\vdash\!X$

Roll:

$$\cancel{t^{01} \to 0 \quad t^{02} \to [t^{02}, t^{01}] \quad t^{12} \to [t^{02}, t^{01}]}$$

$$|X \to |X \cdot (t^{02} - t^{01}) = (t^{01} - t^{02})|X$$

$$t^{01} \to 0 \qquad t^{02} \to [t^{01}, t^{02}] \qquad t^{12} \to -[t^{01}, t^{02}]$$

$$e^{\alpha R}(|X) = \sum \frac{\alpha^n R^n}{n!}(|X) =$$

carol@comp...

$$R^0 |X = |X$$

$$R|X = (t^{01} - t^{02})(|X)$$

$$R^2 |X = \cancel{[t^{01}, t^{02}](t^{01}-t^{02})(|X)} \; (t^{01}-t^{02})^2(|X) - [t^{01}, t^{02}](|X)$$

$$R^3 |X = \cancel{[t^{01}, [t^{01}, t^{02}]] \; [t^{01}, t^{02}](t^{01} - t^{02})(|X)}$$

$$((t^{01}-t^{02})^3 - 3[t^{01}, t^{02}](t^{01}-t^{02})$$