



$$\begin{aligned}
& \longrightarrow \int_{0 \leq t_1 \leq \dots \leq t_6 \leq 1} \frac{d(1-t_1)}{1-t_1} \frac{dt_2}{t_2} \frac{dt_3}{t_3} \frac{d(1-t_4)}{1-t_4} \frac{d(1-t_5)}{1-t_5} \frac{dt_6}{t_6} \\
& = - \int_0^1 \frac{dt_6}{t_6} \int_0^{t_6} \frac{dt_5}{1-t_5} \int_0^{t_5} \frac{dt_4}{1-t_4} \int_0^{t_4} \frac{dt_3}{t_3} \int_0^{t_3} \frac{dt_2}{t_2} \int_0^{t_2} \frac{dt_1}{1-t_1} \\
& \quad \left| \begin{array}{l} \sum_{k_1 > 0} t_1^{k_1-1} \\ \sum_{k_1 > 0} \frac{t_2^{k_1}}{k_1} \\ \sum_{k_1 > 0} \frac{t_3^{k_1}}{k_1^2} \\ \sum_{k_1 > 0} \frac{t_4^{k_1}}{k_1^3} \\ \sum_{k_1, k_2 > 0} \frac{t_5^{k_1+k_2}}{k_1^3(k_1+k_2)} \\ \sum_{k_1, k_2, k_3 > 0} \frac{t_6^{k_1+k_2+k_3}}{k_1^3(k_1+k_2)(k_1+k_2+k_3)} \end{array} \right. \\
& = - \sum_{k_1, k_2, k_3 > 0} \frac{1^{k_1+k_2+k_3}}{k_1^3(k_1+k_2)(k_1+k_2+k_3)^2} \\
& = - \sum_{0 < n_1 < n_2 < n_3} \frac{1}{n_1^3 n_2 n_3^2} =: -\zeta(3, 1, 2).
\end{aligned}$$